Simulation Based on Michel Fodje's epr-simple simulation translated from Python to Mathematica by John Reed 13 Nov 2013 Plus Quaternions Modified by Fred Diether for Completely Local-Realistic Nov. 2021 Includes Joy's S^3 Quaternion Model. With 3D Vectors!

Load Quaternion Package, Set Run Time Parameters, Initialize Arrays and Tables

```
In[1]:= << Quaternions`
β0 = Quaternion[1, 0, 0, 0];
β1 = Quaternion[0, 1, 0, 0];
β2 = Quaternion[0, 0, 1, 0];
β3 = Quaternion[0, 0, 0, 1];
Qcoordinates = {β1, β2, β3};
Qcoordinates2 = {β0, β1, β2, β3};
m = 6 000 000;
trialDeg = 361;
Ls1 = ConstantArray[0, m];
Ls2 = ConstantArray[0, m];
λ1 = ConstantArray[0, m];
λ2 = ConstantArray[0, m];
Daa = ConstantArray[0, m];
Dbb = ConstantArray[0, m];
qA = ConstantArray[0, m];
qB = ConstantArray[0, m];
qa = ConstantArray[0, m];
aq1 = ConstantArray[0, m];
qa1 = ConstantArray[0, m];
qb = ConstantArray[0, m];
bq1 = ConstantArray[0, m];
qb1 = ConstantArray[0, m];
aa1 = ConstantArray[0, m];
bb1 = ConstantArray[0, m];
outA1 = Table[{0, 0}, m];
outA2 = Table[{0, 0}, m];
outB1 = Table[{0, 0}, m];
outB2 = Table[{0, 0}, m];
a1 = ConstantArray[0, m];
b1 = ConstantArray[0, m];
nPP = ConstantArray[0, trialDeg];
nNN = ConstantArray[0, trialDeg];
nPN = ConstantArray[0, trialDeg];
nNP = ConstantArray[0, trialDeg];
nAP = ConstantArray[0, trialDeg];
nBP = ConstantArray[0, trialDeg];
nAN = ConstantArray[0, trialDeg];
nBN = ConstantArray[0, trialDeg];
ϕ = 3; β = 0.284; ξ = 0.892; (*Adustable parameters for fine tuning*)
```

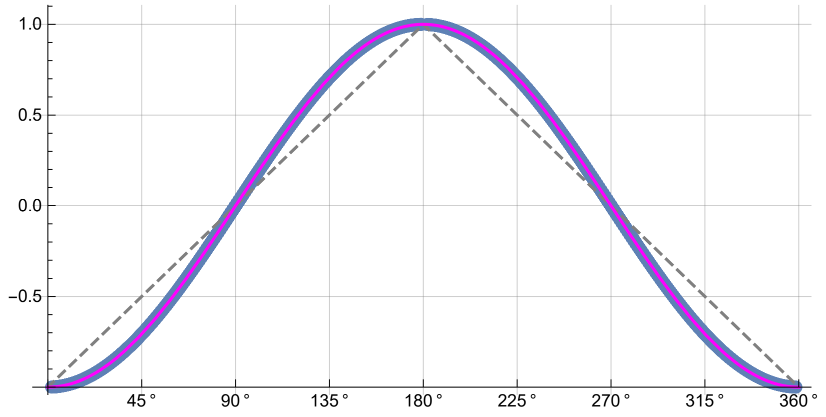
Generating Particle Data with Three Independent Do-Loops

```
In[41]:= Do[θ = RandomPoint[Sphere[]]; (*Singlet 3D vector*) (*Hidden Variable*)
          θ1 = ToSphericalCoordinates[θ][[3]] * 180 / π;
          θ2 = ToSphericalCoordinates[θ][[2]];
          λ1[[i]] = β (Cos[θ1/ϕ]^2);
          λ2[[i]] = (Cos[(θ2 * ξ)/2]^2);
          Ls1[[i]] = θ.Qcoordinates; (*Convert to quaternion coordinates*)
          Ls2[[i]] = -θ.Qcoordinates, {i, m}]

In[42]:= Do[a = RandomPoint[Sphere[]]; (*Detector 3D vector angle*)
          aa1[[i]] = a;
          Da = a.Qcoordinates; (*Convert to quaternion coordinates*)
          Daa[[i]] = Da;
          qa = Da ** Ls1[[i]];
          qa1[[i]] = qa;
          aq = -Da ** Ls1[[i]];
          aq1[[i]] = aq;
          If[Abs[Re[qa]] > λ1[[i]],
           qA1 = Re[Da ** Limit[Ls1[[i]], Ls1[[i]] → Sign[Re[Da ** Ls1[[i]]]] Da]],
           qA1 = Sign[aq[[4]]] + 0.001];
          outA1[[i]] = {a, qA1};
          If[Abs[Re[qa]] > λ2[[i]],
           qA2 = Re[Da ** Limit[Ls1[[i]], Ls1[[i]] → Sign[Re[Da ** Ls1[[i]]]] Da]],
           qA2 = Sign[aq[[4]]] + 0.001];
          outA2[[i]] = {a, qA2}, {i, m}]
         outA = Catenate[{outA1, outA2}];

In[44]:= Do[b = RandomPoint[Sphere[]]; (*Detector 3D vector angle*)
          bb1[[i]] = b;
          Db = b.Qcoordinates; (*Convert to quaternion coordinates*)
          Dbb[[i]] = Db;
          qb = Ls2[[i]] ** Db;
          qb1[[i]] = qb;
          bq = -Ls2[[i]] ** Db;
          bq1[[i]] = bq;
          If[Abs[Re[qb]] > λ1[[i]],
           qB1 = Re[Db ** Limit[Ls2[[i]], Ls2[[i]] → Sign[Re[Db ** Ls2[[i]]]] Db]],
           qB1 = Sign[bq[[4]]] + 0.001];
          outB1[[i]] = {b, qB1};
          If[Abs[Re[qb]] > λ2[[i]],
           qB2 = Re[Db ** Limit[Ls2[[i]], Ls2[[i]] → Sign[Re[Db ** Ls2[[i]]]] Db]],
           qB2 = Sign[bq[[4]]] + 0.001];
          outB2[[i]] = {b, qB2}, {i, m}]
         outB = Catenate[{outB1, outB2}];
```

Verification of the Analytical 3-Sphere Model Based on Geometric Algebra using Quaternions

```
In[123]:= q = 0; s = 0;
          m3 = 20 000;
          DA = Take[Daa, m3];
          DB = Take[Dbb, m3];
          Ls11 = Take[Ls1, m3];
          Ls22 = Take[Ls2, m3];
          QA = DA ** Ls11;
          QB = Ls22 ** DB;
          r0 = ConstantArray[0, m3];
          r1 = ConstantArray[0, m3];
          r2 = ConstantArray[0, m3];
          QA1 = ConstantArray[0, m3];
          QB1 = ConstantArray[0, m3];
          QAB = ConstantArray[0, m3];
          angle = ConstantArray[0, m3];
          plotq = Table[{0, 0}, m3];
          (*qA=Re[DA**Limit[Ls11[[i]],Ls11[[i]]→Sign[Re[DA**Ls11[[i]]]]DA]];
            qB=Re[DB**Limit[Ls22[[i]],Ls22[[i]]→Sign[Re[DB[[i]]**Ls22[[i]]]]DB]];*)
          (*These two lines moved to the A and B Do-loops
            for further proper local processing*)
```

```
In[139]:= Do[r1[[i]] = ({QA[[i]][[2]], QA[[i]][[3]], QA[[i]][[4]]});
  r2[[i]] = ({QB[[i]][[2]], QB[[i]][[3]], QB[[i]][[4]]});
  QA1[[i]] = Flatten[{Re[QA[[i]]], r1[[i]]}].Qcoordinates2;
  QB1[[i]] = Flatten[{Re[QB[[i]]], r2[[i]]}].Qcoordinates2;
  QAB[[i]] = Re[QA1[[i]] ** QB1[[i]]];
 r0[[i]] = (Re[qa1[[i]]] Limit[Cross[θ3, bb1[[i]]], θ3 → Sign[Re[qb1[[i]]]] bb1[[i]]] +
      Re[qb1[[i]]] Limit[Cross[aa1[[i]], θ4], θ4 → Sign[Re[qa1[[i]]]] aa1[[i]]] -
      Limit[Cross[aa1[[i]], θ5], θ5 → Sign[Re[qa1[[i]]]] aa1[[i]]] *
       Limit[Cross[θ6, bb1[[i]]], θ6 → Sign[Re[qb1[[i]]]] bb1[[i]]]) /
    (Sin[ArcCos[aa1[[i]].bb1[[i]]]]);
  q = {Re[QAB[[i]]], r0[[i]][[1]], r0[[i]][[2]], r0[[i]][[3]]}.Qcoordinates2;
  ϕA = ArcTan[aa1[[i]][[1]], aa1[[i]][[2]]] / 50;
  ϕB = ArcTan[bb1[[i]][[2]], bb1[[i]][[1]]] / 50;
  If[ϕA * ϕB > 0, angle = ArcCos[aa1[[i]].bb1[[i]]] * 180 / π,
   angle = (2 π - ArcCos[aa1[[i]].bb1[[i]]]) * 180 / π];
  s = s + q[[1]];
  plotq[[i]] = {angle, q[[1]]}, {i, m3}]
 Meanq = s / m; (*shows vanishing of the non-real part iJK*)
 Print["Meanq = ", Meanq]
 sim = ListPlot[plotq, PlotMarkers → {Automatic, Small},
    AspectRatio → 8 / 16, Ticks → {{{0, 0 °}, {45, 45 °}, {90, 90 °}, {135, 135 °},
       {180, 180 °}, {225, 225 °}, {270, 270 °}, {315, 315 °}, {360, 360 °}}, Automatic},
    GridLines → Automatic, AxesOrigin → {0, -1.0}];
 p1 = Plot[-1 + 2 x Degree / π, {x, 0, 180}, PlotStyle → {Gray, Dashed}];
 p2 = Plot[3 - 2 x Degree / π, {x, 180, 360}, PlotStyle → {Gray, Dashed}];
 negcos1 = Plot[-Cos[x Degree], {x, 0, 360}, PlotStyle → {Magenta}];
 Show[sim, p1, p2, negcos1]

 Meanq = 9.16956×10^-6
```

Out[146]=


Blue is the data, magenta is the negative cosine curve for an exact match.

Statistical Analysis of the Particle Data Received from Alice and Bob

```
In[70]:= m2 = 2 m;
         theta = ConstantArray[0, m2];
         th1 = ConstantArray[0, m2];
         a1 = outA[[All, 1]];
         qA = outA[[All, 2]];
         b1 = outB[[All, 1]];
         qB = outB[[All, 2]];
         Do[ϕA1 = ArcTan[a1[[i]][[1]], a1[[i]][[2]]] / 50;
          ϕB1 = ArcTan[b1[[i]][[2]], b1[[i]][[1]]] / 50;
          If[ϕA1 * ϕB1 > 0, th1[[i]] = ArcCos[a1[[i]].b1[[i]]],
           th1[[i]] = 2 π - ArcCos[a1[[i]].b1[[i]]]];
          theta[[i]] = Round[th1[[i]] * 180 / π] + 1;
          th = theta[[i]];
          aliceD = qA[[i]]; bobD = qB[[i]];
          If[aliceD == 1, nAP[[th]] ++];
          If[bobD == 1, nBP[[th]] ++];
          If[aliceD == -1, nAN[[th]] ++];
          If[bobD == -1, nBN[[th]] ++];
          If[aliceD == 1 && bobD == 1, nPP[[th]] ++];
          If[aliceD == 1 && bobD == -1, nPN[[th]] ++];
          If[aliceD == -1 && bobD == 1, nNP[[th]] ++];
          If[aliceD == -1 && bobD == -1, nNN[[th]] ++], {i, m2}]
```

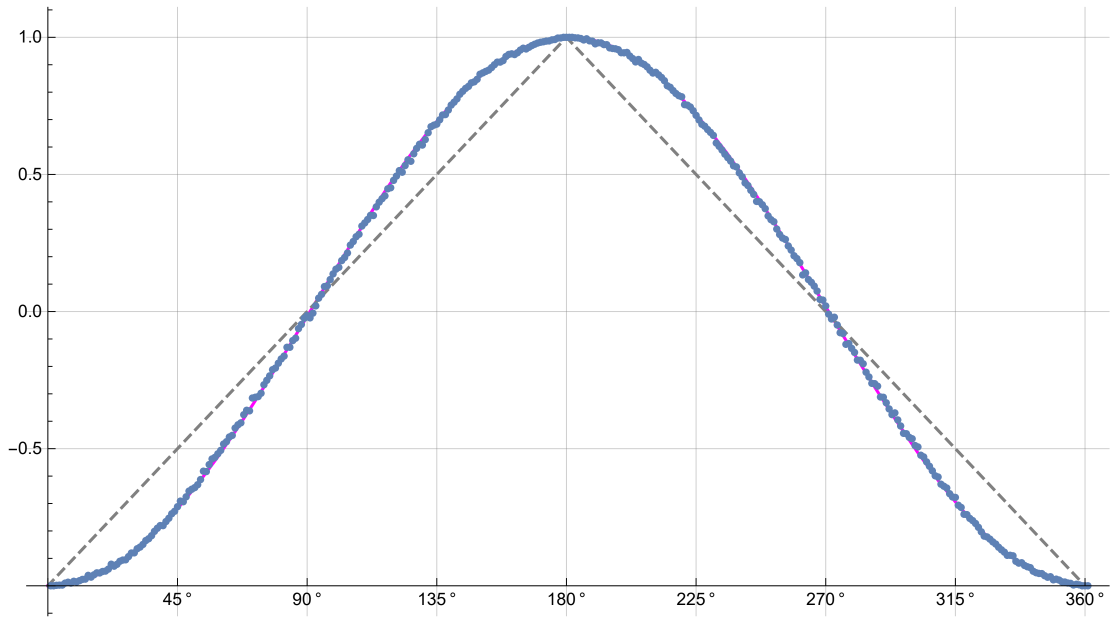
Calculating Mean Values of AB

```
In[75]:= mean = ConstantArray[0, trialDeg];
         sum1 = ConstantArray[0, trialDeg];
         sum2 = ConstantArray[0, trialDeg];
         Do[sum1[[i]] = (nPP[[i]] + nNN[[i]] - nPN[[i]] - nNP[[i]]);
          sum2[[i]] = nPP[[i]] + nPN[[i]] + nNP[[i]] + nNN[[i]] + 0.0000001;
         mean[[i]] = sum1[[i]] / sum2[[i]], {i, trialDeg}]
```

Plotting the Results and Comparing Mean Values with -Cosine Function

```
In[79]:= simulation = ListPlot[mean, PlotMarkers → {Automatic, Tiny}];
         negcos =
           Plot[-Cos[x Degree - 1 Degree], {x, 0, 361}, PlotStyle → {Magenta}, AspectRatio → 9/16, Ticks →
             {{{0, 0 °}, {45, 45 °}, {90, 90 °}, {135, 135 °}, {180, 180 °}, {225, 225 °}, {270, 270 °},
               {315, 315 °}, {360, 360 °}}, Automatic}, GridLines → Automatic, AxesOrigin → {0, -1.0}];
         p1 = Plot[-1 + 2 x Degree / π, {x, 0, 180}, PlotStyle → {Gray, Dashed}];
         p2 = Plot[3 - 2 x Degree / π, {x, 180, 360}, PlotStyle → {Gray, Dashed}];
         Show[negcos, p1, p2, simulation]
```

Out[83]=



Computing Averages

```
In[84]:= A1 = ConstantArray[0, m2];
B1 = ConstantArray[0, m2];
Do[If[qA[[i]] == 1 || qA[[i]] == -1, A1[[i]] = qA[[i]]];
 If[qB[[i]] == 1 || qB[[i]] == -1, B1[[i]] = qB[[i]]], {i, m2}]
AveA = N[Sum[A1[[i]], {i, m2}] / m2];
AveB = N[Sum[B1[[i]], {i, m2}] / m2];
Print["AveA = ", AveA];
Print["AveB = ", AveB];
PAP = N[Sum[nAP[[i]], {i, trialDeg}]];
PBP = N[Sum[nBP[[i]], {i, trialDeg}]];
PAN = N[Sum[nAN[[i]], {i, trialDeg}]];
PBN = N[Sum[nBN[[i]], {i, trialDeg}]];
PA1 = PAP / (PAP + PAN);
PB1 = PBP / (PBP + PBN);
Print["P(A+)= ", PA1]
Print["P(B+)= ", PB1]
totAB = Sum[nPP[[i]] + nNN[[i]] + nPN[[i]] + nNP[[i]], {i, trialDeg}];
Print["Total Events = ", totAB]
PP = N[Sum[nPP[[i]], {i, trialDeg}] / totAB];
NN = N[Sum[nNN[[i]], {i, trialDeg}] / totAB];
PN = N[Sum[nPN[[i]], {i, trialDeg}] / totAB];
NP = N[Sum[nNP[[i]], {i, trialDeg}] / totAB];
totP = PP + NN + PN + NP;
Print["Ave ++ = ", PP]
Print["Ave -- = ", NN]
Print["Ave +- = ", PN]
Print["Ave -+ = ", NP]
CHSH = Abs[N[mean[[23]]] - N[mean[[135]]] + N[mean[[68]]] + N[mean[[45]]]];
Print["Approx. CHSH = ", CHSH]

AveA = 0.000480083

AveB = 0.0000890833

P(A+)= 0.500373

P(B+)= 0.500069

Total Events = 5994562

Ave ++ = 0.250082

Ave -- = 0.249722

Ave +- = 0.250138

Ave -+ = 0.250058

Approx. CHSH = 2.69972
```

In[112]:= $E_{ab} = \text{TrigReduce}\left[\frac{\text{Sin}[(\eta_{ab})/2]^2}{2} + \frac{\text{Sin}[(\eta_{ab})/2]^2}{2} - \frac{\text{Cos}[(\eta_{ab})/2]^2}{2} - \frac{\text{Cos}[(\eta_{ab})/2]^2}{2}\right] / \text{totP};$

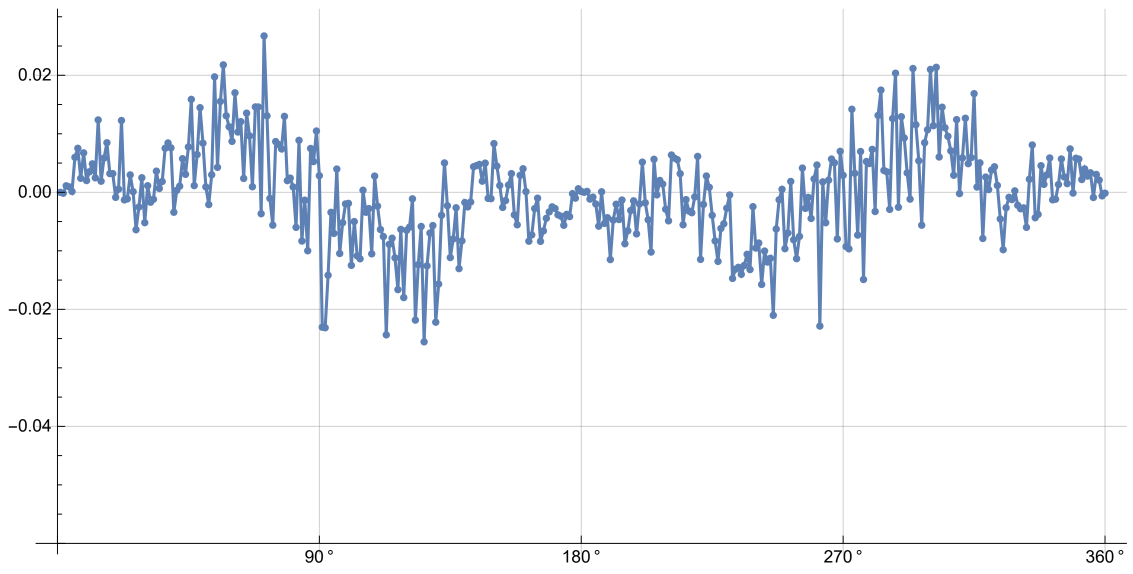
Print["E(a, b) = ", E_{ab}]

E(a, b) = $-1.\ \text{Cos}[\eta_{ab}]$

Calculating Deviation from -Cosine Curve

```
In[114]:= dev1 = ConstantArray[2, 360];
          dev2 = ConstantArray[2, 360];
          dev3 = ConstantArray[2, 360];
          Do[dev1 = mean[[i]];
           dev2[[i]] = {dev1, i}, {i, 360}]
          devang = dev2[[All, 2]];
          Do[dev3[[i]] = mean[[i]] + Cos[devang[[i]] Degree - 1 Degree], {i, 360}]
          ListPlot[N[dev3], PlotMarkers → {Automatic, Tiny}, Joined → True, AspectRatio → 1/2,
           Ticks → {{{0, 0 °}, {90, 90 °}, {180, 180 °}, {270, 270 °}, {360, 360 °}}, Automatic},
           GridLines → Automatic, AxesOrigin → {0, -0.06}]
```

Out[120]=



```
In[121]:= N[Mean[Abs[dev3]]]
          N[Mean[dev3]]

Out[121]= 0.00622659

Out[122]= -0.000176737
```