

**Simulation Based on Michel Fodje's epr-simple simulation translated from Python to Mathematica by John Reed 13 Nov 2013 Plus Quaternions Modified by Fred Diether for Completely Local-Realistic Dec. 2021 Using Joy's  $S^3$  Quaternion Model. With 3D Vectors!**

**Load Quaternion Package, Set Run Time Parameters, Initialize Arrays and Tables**

```
In[1538]:= << Quaternions`
 $\beta_0$  = Quaternion[1, 0, 0, 0];
 $\beta_1$  = Quaternion[0, 1, 0, 0];
 $\beta_2$  = Quaternion[0, 0, 1, 0];
 $\beta_3$  = Quaternion[0, 0, 0, 1];
Qcoordinates = { $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ };
Qcoordinates2 = { $\beta_0$ ,  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$ };
m = 1000000;
trialDeg = 361;
ss = ConstantArray[0, m];
Ls1 = ConstantArray[0, m];
Ls2 = ConstantArray[0, m];
 $\lambda_1$  = ConstantArray[0, m];
 $\lambda_2$  = ConstantArray[0, m];
Da1 = ConstantArray[0, m];
Db1 = ConstantArray[0, m];
qA = ConstantArray[0, m];
qB = ConstantArray[0, m];
qa = ConstantArray[0, m];
qb = ConstantArray[0, m];
aq1 = ConstantArray[0, m];
qa1 = ConstantArray[0, m];
bq1 = ConstantArray[0, m];
qb1 = ConstantArray[0, m];
aa1 = ConstantArray[0, m];
bb1 = ConstantArray[0, m];
outA1 = Table[{0, 0}, m];
outA2 = Table[{0, 0}, m];
outB1 = Table[{0, 0}, m];
outB2 = Table[{0, 0}, m];
a1 = ConstantArray[0, m];
b1 = ConstantArray[0, m];
nPP = ConstantArray[0, trialDeg];
nNN = ConstantArray[0, trialDeg];
nPN = ConstantArray[0, trialDeg];
nNP = ConstantArray[0, trialDeg];
nAP = ConstantArray[0, trialDeg];
nBP = ConstantArray[0, trialDeg];
nAN = ConstantArray[0, trialDeg];
nBN = ConstantArray[0, trialDeg];
 $\phi$  = 3;  $\beta$  = 0.284;  $\xi$  = 0.892; (*Adjustable parameters for fine tuning*)
```

**Generating Particle Data with Three Independent Do-Loops**

```
In[1579]:= Do[s = RandomPoint[Sphere[]]; (*Singlet 3D vector*) (*Hidden Variable*)
  ss[[i]] = s;
   $\theta_1 = \text{ToSphericalCoordinates}[s][[3]] * 180 / \pi;$ 
   $\theta_2 = \text{ToSphericalCoordinates}[s][[2]];$ 
   $\lambda_1[[i]] = \beta \left( \cos\left[\frac{\theta_1}{\phi}\right]^2 \right);$ 
   $\lambda_2[[i]] = \left( \cos\left[\frac{\theta_2 * \xi}{2}\right]^2 \right);$ 
  Ls1[[i]] = s.Qcoordinates; (*Convert to quaternion coordinates*)
  Ls2[[i]] = -s.Qcoordinates, {i, m}]
```

```
In[1580]:= Do[a = RandomPoint[Sphere[]]; (*Detector 3D vector angle*)
  aa1[[i]] = a;
  Da = a.Qcoordinates; (*Convert to quaternion coordinates*)
  Da1[[i]] = Da;
  qa = Da ** Ls1[[i]];
  qa1[[i]] = qa;
  aq = -Da ** Ls1[[i]];
  aq1[[i]] = aq;
  If[Abs[Re[qa]] >  $\lambda_1[[i]]$ ,
    qA1 = Re[Da ** Limit[Ls1[[i]], Ls1[[i]] → Sign[Re[Da ** Ls1[[i]]]]] Da],
    qA1 = Sign[{aq[[2]], aq[[3]], aq[[4]]}.Qcoordinates]];
  outA1[[i]] = {a, qA1};
  If[Abs[Re[qa]] >  $\lambda_2[[i]]$ ,
    qA2 = Re[-Da ** Limit[Ls1[[i]], Ls1[[i]] → Sign[Re[Da ** Ls1[[i]]]]] Da],
    qA2 = Sign[{qa[[2]], qa[[3]], qa[[4]]}.Qcoordinates]];
  outA2[[i]] = {a, qA2}, {i, m}]
  outA = Catenate[{outA1, outA2}];
```

```
In[1582]:= Do[b = RandomPoint[Sphere[]]; (*Detector 3D vector angle*)
  bb1[[i]] = b;
  Db = b.Qcoordinates; (*Convert to quaternion coordinates*)
  Db1[[i]] = Db;
  qb = Ls2[[i]] ** Db;
  qb1[[i]] = qb;
  bq = -Ls2[[i]] ** Db;
  bq1[[i]] = bq;
  If[Abs[Re[qb]] >  $\lambda_1[[i]]$ ,
    qB1 = Re[Db ** Limit[Ls2[[i]], Ls2[[i]] → Sign[Re[Db ** Ls2[[i]]]]] Db],
    qB1 = Sign[{bq[[2]], bq[[3]], bq[[4]]}.Qcoordinates]];
  outB1[[i]] = {b, qB1};
  If[Abs[Re[qb]] >  $\lambda_2[[i]]$ ,
    qB2 = Re[-Db ** Limit[Ls2[[i]], Ls2[[i]] → Sign[Re[Db ** Ls2[[i]]]]] Db],
    qB2 = Sign[{qb[[2]], qb[[3]], qb[[4]]}.Qcoordinates]];
  outB2[[i]] = {b, qB2}, {i, m}]
  outB = Catenate[{outB1, outB2}];
```

## Product Calculation Verification

```

In[1584]:= m3 = 20000;
m4 = m3 * 2;
aa2 = Take[aa1, m3];
bb2 = Take[bb1, m3];
DA = Take[Da1, m3];
DB = Take[Db1, m3];
λ3 = Take[λ1, m3];
λ4 = Take[λ2, m3];
Ls11 = Take[Ls1, m3];
Ls22 = Take[Ls2, m3];
QA = DA ** Ls11;
QB = Ls22 ** DB;
AQ = -DA ** Ls11;
BQ = -Ls22 ** DB;
outA11 = Table[{0, 0, 0}, m3];
outA22 = Table[{0, 0, 0}, m3];
outB11 = Table[{0, 0, 0}, m3];
outB22 = Table[{0, 0, 0}, m3];
r0 = ConstantArray[0, m4];
r1 = ConstantArray[0, m4];
r2 = ConstantArray[0, m4];
qAB = ConstantArray[0, m4];
angleθ = ConstantArray[0, m4];
AB6 = ConstantArray[0, m4];
QAB = Table[{0, 0}, m4];

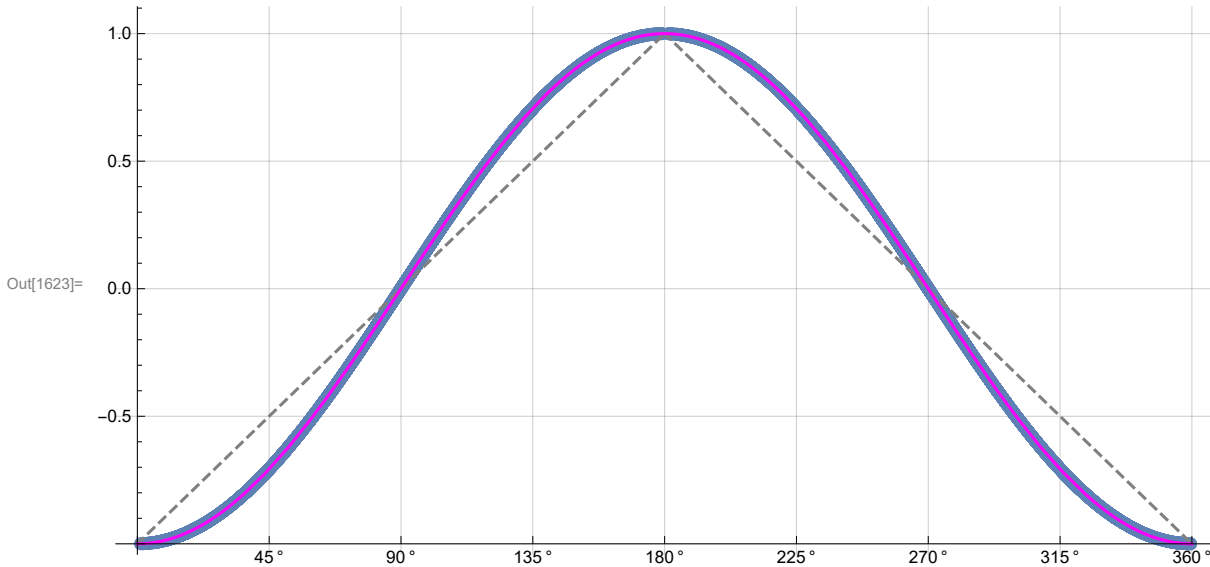
In[1609]:= Do[If[Abs[Re[QA[[i]]]] > λ3[[i]], qA1 = Re[QA[[i]]],
  qA1 = {AQ[[i]][[2]], AQ[[i]][[3]], AQ[[i]][[4]]}.Qcoordinates];
  If[Abs[Re[QA[[i]]]] > λ3[[i]], ra1 = {QA[[i]][[2]], QA[[i]][[3]], QA[[i]][[4]]},
  ra1 = {AQ[[i]][[2]], AQ[[i]][[3]], AQ[[i]][[4]]}.Qcoordinates];
  outA11[[i]] = {aa2[[i]], qA1, ra1};
  If[Abs[Re[QA[[i]]]] > λ4[[i]], qA2 = Re[-QA[[i]]],
  qA2 = {QA[[i]][[2]], QA[[i]][[3]], QA[[i]][[4]]}.Qcoordinates];
  If[Abs[Re[QA[[i]]]] > λ4[[i]], ra2 = {AQ[[i]][[2]], AQ[[i]][[3]], AQ[[i]][[4]]},
  ra2 = {QA[[i]][[2]], QA[[i]][[3]], QA[[i]][[4]]}.Qcoordinates];
  outA22[[i]] = {aa2[[i]], qA2, ra2};
  If[Abs[Re[QB[[i]]]] > λ3[[i]], qB11 = Re[QB[[i]]],
  qB11 = {BQ[[i]][[2]], BQ[[i]][[3]], BQ[[i]][[4]]}.Qcoordinates];
  If[Abs[Re[QB[[i]]]] > λ3[[i]], rb1 = {QB[[i]][[2]], QB[[i]][[3]], QB[[i]][[4]]},
  rb1 = {BQ[[i]][[2]], BQ[[i]][[3]], BQ[[i]][[4]]}.Qcoordinates];
  outB11[[i]] = {bb2[[i]], qB11, rb1};
  If[Abs[Re[QB[[i]]]] > λ4[[i]], qB22 = Re[-QB[[i]]],
  qB22 = {QB[[i]][[2]], QB[[i]][[3]], QB[[i]][[4]]}.Qcoordinates];
  If[Abs[Re[QB[[i]]]] > λ4[[i]], rb2 = {BQ[[i]][[2]], BQ[[i]][[3]], BQ[[i]][[4]]},
  rb2 = {QB[[i]][[2]], QB[[i]][[3]], QB[[i]][[4]]}.Qcoordinates];
  outB22[[i]] = {bb2[[i]], qB22, rb2}, {i, m3}]

In[1610]:= outAq = Catenate[{outA11, outA22}];
outBq = Catenate[{outB11, outB22}];
a6 = outAq[[A11, 1]];
b6 = outBq[[A11, 1]];
A6 = outAq[[A11, 2]];
B6 = outBq[[A11, 2]];
ra12 = outAq[[A11, 3]];
rb12 = outBq[[A11, 3]];

```

```
In[1618]:= Do[r1[[i]] = ra12[[i]];
r2[[i]] = rb12[[i]];
qAB[[i]] = Re[A6[[i]]] * Re[B6[[i]]] - (r1[[i]].r2[[i]]);
r0[[i]] = (Re[qa1[[i]]] Limit[Cross[s2, bb1[[i]]], s2 → Sign[Re[qb1[[i]]]] bb1[[i]] +
Re[qb1[[i]]] Limit[Cross[aa1[[i]], s1], s1 → Sign[Re[qa1[[i]]]] aa1[[i]] -
Limit[Cross[aa1[[i]], s1], s1 → Sign[Re[qa1[[i]]]] aa1[[i]] *
Limit[Cross[s2, bb1[[i]]], s2 → Sign[Re[qb1[[i]]]] bb1[[i]]) /
(Sin[ArcCos[aa1[[i]].bb1[[i]]]));
phiA1 = ArcTan[a6[[i]][[1]], a6[[i]][[2]]] / 50;
phiB1 = ArcTan[b6[[i]][[2]], b6[[i]][[1]]] / 50;
If[phiA1 * phiB1 > 0, angleTheta[[i]] = ArcCos[a6[[i]].b6[[i]]] * 180 / Pi,
angleTheta[[i]] = (2 Pi - ArcCos[a6[[i]].b6[[i]]) * 180 / Pi];
q = {qAB[[i]], r0[[i]][[1]], r0[[i]][[2]], r0[[i]][[3]]}.Qcoordinates2;
QAB[[i]] = {angleTheta[[i]], q[[1]]}, {i, m4}]
```

```
In[1619]:= qsim2 = ListPlot[QAB, PlotMarkers → {Automatic, Small}, AspectRatio → 8 / 16, Ticks →
{{{0, 0°}, {45, 45°}, {90, 90°}, {135, 135°}, {180, 180°}, {225, 225°}, {270, 270°},
{315, 315°}, {360, 360°}}, Automatic, GridLines → Automatic, AxesOrigin → {0, -1.0}];
p1 = Plot[-1 + 2 x Degree / Pi, {x, 0, 180}, PlotStyle → {Gray, Dashed}];
p2 = Plot[3 - 2 x Degree / Pi, {x, 180, 360}, PlotStyle → {Gray, Dashed}];
negcos2 = Plot[-Cos[x Degree], {x, 0, 360}, PlotStyle → {Magenta}];
Show[qsim2, p1, p2, negcos2]
```



Blue is data and magenta is the -cosine curve for an exact match!

### Statistical Analysis of the Particle Data Received from Alice and Bob

```

In[1624]:= m2 = 2 m;
theta = ConstantArray[0, m2];
th = ConstantArray[0, m2];
a1 = outA[[All, 1]];
qA = outA[[All, 2]];
b1 = outB[[All, 1]];
qB = outB[[All, 2]];
Do[phiA1 = ArcTan[a1[[i]][[1]], a1[[i]][[2]]] / 50;
  phiB1 = ArcTan[b1[[i]][[2]], b1[[i]][[1]]] / 50;
  If[phiA1 * phiB1 > 0, th[[i]] = ArcCos[a1[[i]].b1[[i]]],
    th[[i]] = 2 pi - ArcCos[a1[[i]].b1[[i]]];
  theta[[i]] = Round[th[[i]] * 180 / pi] + 1;
  theta = theta[[i]];
  aliceD = qA[[i]]; bobD = qB[[i]];
  If[aliceD == 1, nAP[[theta]] ++];
  If[bobD == 1, nBP[[theta]] ++];
  If[aliceD == -1, nAN[[theta]] ++];
  If[bobD == -1, nBN[[theta]] ++];
  If[aliceD == 1 && bobD == 1, nPP[[theta]] ++];
  If[aliceD == 1 && bobD == -1, nPN[[theta]] ++];
  If[aliceD == -1 && bobD == 1, nNP[[theta]] ++];
  If[aliceD == -1 && bobD == -1, nNN[[theta]] ++], {i, m2}]

```

## Calculating Mean Values of AB

```

In[1629]:= mean = ConstantArray[0, trialDeg];
sum1 = ConstantArray[0, trialDeg];
sum2 = ConstantArray[0, trialDeg];
Do[sum1[[i]] = (nPP[[i]] + nNN[[i]] - nPN[[i]] - nNP[[i]]);
  sum2[[i]] = nPP[[i]] + nPN[[i]] + nNP[[i]] + nNN[[i]] + 0.0000001;
  mean[[i]] = sum1[[i]] / sum2[[i]], {i, trialDeg}]

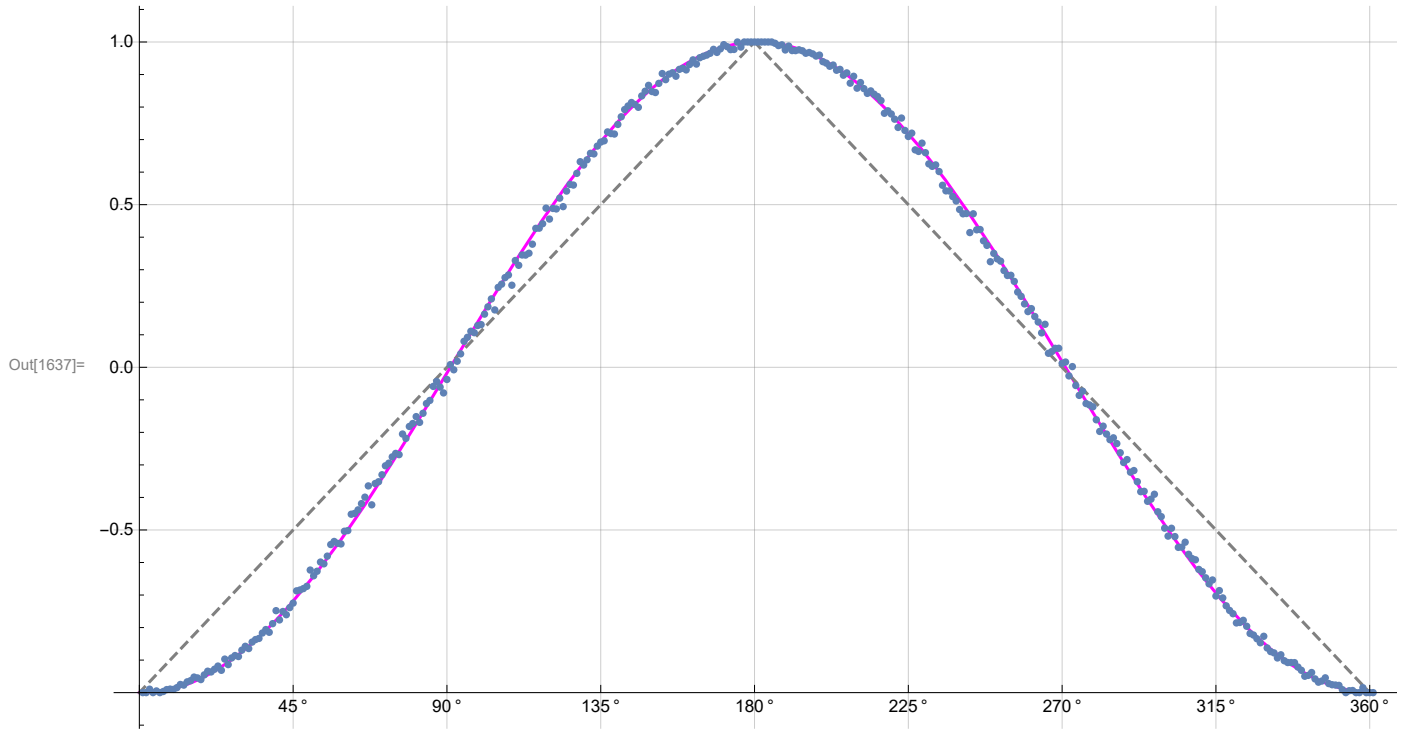
```

## Plotting the Results and Comparing Mean Values with -Cosine Function

```

In[1633]:= simulation = ListPlot[mean, PlotMarkers → {Automatic, Tiny}];
negcos =
  Plot[-Cos[x Degree - 1 Degree], {x, 0, 361}, PlotStyle → {Magenta}, AspectRatio → 9/16, Ticks →
    {{0, 0°}, {45, 45°}, {90, 90°}, {135, 135°}, {180, 180°}, {225, 225°}, {270, 270°},
     {315, 315°}, {360, 360°}}, Automatic, GridLines → Automatic, AxesOrigin → {0, -1.0}];
p1 = Plot[-1 + 2 x Degree / π, {x, 0, 180}, PlotStyle → {Gray, Dashed}];
p2 = Plot[3 - 2 x Degree / π, {x, 180, 360}, PlotStyle → {Gray, Dashed}];
Show[negcos, p1, p2, simulation]

```



## Computing Averages

```

In[1638]:= A1 = ConstantArray[0, m2];
B1 = ConstantArray[0, m2];
Do[If[qA[[i]] == 1 || qA[[i]] == -1, A1[[i]] = qA[[i]];
  If[qB[[i]] == 1 || qB[[i]] == -1, B1[[i]] = qB[[i]], {i, m2}];
AveA = N[Sum[A1[[i]], {i, m2}]/m2];
AveB = N[Sum[B1[[i]], {i, m2}]/m2];
Print["AveA = ", AveA];
Print["AveB = ", AveB];
PAP = N[Sum[nAP[[i]], {i, trialDeg}]];
PBP = N[Sum[nBP[[i]], {i, trialDeg}]];
PAN = N[Sum[nAN[[i]], {i, trialDeg}]];
PBN = N[Sum[nBN[[i]], {i, trialDeg}]];
PA1 = PAP / (PAP + PAN);
PB1 = PBP / (PBP + PBN);
Print["P(A+) = ", PA1]
Print["P(B+) = ", PB1]
totAB = Total[nPP + nNN + nPN + nNP];
Print["Total Events Detected = ", totAB]
PP = N[Sum[nPP[[i]], {i, trialDeg}]/totAB];
NN = N[Sum[nNN[[i]], {i, trialDeg}]/totAB];
PN = N[Sum[nPN[[i]], {i, trialDeg}]/totAB];
NP = N[Sum[nNP[[i]], {i, trialDeg}]/totAB];
totP = PP + NN + PN + NP;
Print["Ave ++ = ", PP]
Print["Ave -- = ", NN]
Print["Ave +- = ", PN]
Print["Ave -+ = ", NP]
CHSH = Abs[N[mean[[23]]] - N[mean[[135]]] + N[mean[[68]]] + N[mean[[45]]]];
Print["Approx. CHSH = ", CHSH]

AveA = 0.0004655
AveB = -0.0007665
P(A+) = 0.500361
P(B+) = 0.499404
Total Events Detected = 999740
Ave ++ = 0.24952
Ave -- = 0.249736
Ave +- = 0.251012
Ave -+ = 0.249732
Approx. CHSH = 2.75828

In[1666]:= Eab = TrigReduce[
$$\frac{\sin[(\eta_{ab})/2]^2}{2} + \frac{\sin[(\eta_{ab})/2]^2}{2} - \frac{\cos[(\eta_{ab})/2]^2}{2} - \frac{\cos[(\eta_{ab})/2]^2}{2}] / \text{totP};
Print["E(a, b) = ", Eab]
E(a, b) = -1. Cos[\eta_{ab}]$$

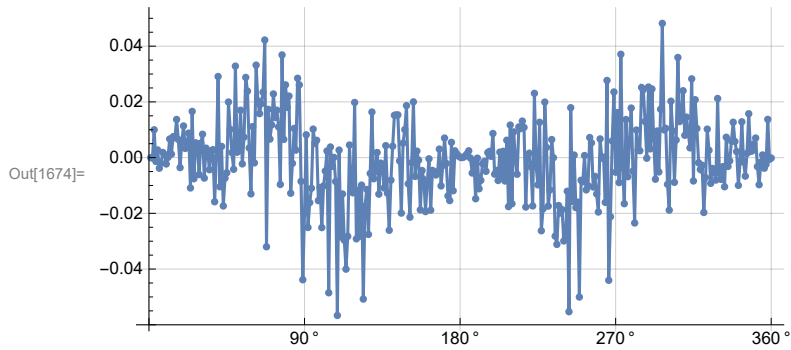
```

## Calculating Deviation from -Cosine Curve

```

In[1668]:= dev1 = ConstantArray[2, 360];
dev2 = ConstantArray[2, 360];
dev3 = ConstantArray[2, 360];
Do[dev1 = mean[[i]];
  dev2[[i]] = {dev1, i}, {i, 360}]
devang = dev2[[All, 2]];
Do[dev3[[i]] = mean[[i]] + Cos[devang[[i]] Degree - 1 Degree], {i, 360}]
ListPlot[N[dev3], PlotMarkers -> {Automatic, Tiny}, Joined -> True, AspectRatio -> 1/2,
  Ticks -> {{{0, 0°}, {90, 90°}, {180, 180°}, {270, 270°}, {360, 360°}}, Automatic},
  GridLines -> Automatic, AxesOrigin -> {0, -0.06}]

```



```

In[1675]:= N[Mean[Abs[dev3]]]
N[Mean[dev3]]

```

Out[1675]= 0.0113525

Out[1676]= -0.000728585