What is the Missing Part of Electron Theory? Gravitational Torsion

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Abstract

We demonstrate that quantum (and classical) electrodynamics can be completed via gravitational torsion using Einstein-Cartan-Sciama-Kibble (ECSK) gravity theory which provides the missing part of electron theory. When using the Dirac type Hehl-Datta equation, another term appears in the Lagrangian for quantum electrodynamics. That term relates the charged fermion spin to gravitational torsion and provides a mechanical energy counter balance to the so-called infinite electromagnetic self-energy. There is no "bare" mass for an electron nor is renormalization required for many scenarios.

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I. INTRODUCTION

Gravitational torsion is the twisting of spacetime. If spacetime can be "bent" or curved as in normal gravity it stands to reason that it should also be able to be twisted. The ECSK gravity theory [1–4] that we will use in our demonstration has been thoroughly researched and is a minimal extension to general relativity that includes the twisting or torsion. For a recent very detailed review see [5]. According to ECSK gravity theory, the spin of a fermion is a mechanism that twists spacetime quite a lot. In fact, a fermion would not exist without the extreme twisting of spacetime for that is what we think it is; twisted spacetime [8].

If one takes the standard Lagrangian for quantum electrodynamics (QED) and solves it for the radius of an electron, the result is the classical electron radius. Which it is known that an electron must be much smaller than that from scattering and other experiments [9]. We have the standard QED Lagrangian density as,

$$\mathfrak{L}_{\mathrm{QED}} = i\hbar c\,\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + e\,\bar{\psi}\gamma^{\mu}A_{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - mc^2\bar{\psi}\psi, \tag{1}$$

then set to zero and go to the rest frame of the electron, we have,

$$+e\,\bar{\psi}\gamma^{0}A_{0}\psi = mc^2\bar{\psi}\psi \;\longrightarrow\; r = \frac{\alpha\hbar}{m_e c}. \tag{2}$$

So, that is a clue that QED is not complete. However, if one uses a Lagrangian from which the Dirac-Hehl-Datta equation (3) is extracted [6] that has the gravitational spin-torsion term in it and solves for the radius, the result is two solutions. One for the classical radius and another radius close to Plank length. So, why would an electron have two "sizes"? We will try to answer that question in this essay.

II. THE GRAVITATIONAL SPIN-TORSION TERM

In 1971, Hehl and Datta were able to formulate a Dirac type of equation [6] with a gravitational spin-torsion term derived from ECSK gravitational theory. We will call it the Hehl-Data equation and it looks like this,

$$\left[\gamma^{\alpha}\nabla^{\{\}}_{\alpha} - \frac{3}{8}il^2(\psi^{+}\gamma_5\gamma^{\alpha}\psi)\gamma_5\gamma_{\alpha}\right]\psi = im\psi, \tag{3}$$

where $l = \sqrt{G\hbar/c^3}$ is Planck length. However, when we multiply through this equation by $-i$ we obtain

$$\left[-i\gamma^{\alpha}\nabla^{\{\}}_{\alpha} - \frac{3}{8}l^2(\psi^{+}\gamma_5\gamma^{\alpha}\psi)\gamma_5\gamma_{\alpha}\right]\psi = m\psi\,. \tag{4}$$

We think the negative sign on the kinetic energy term is due to the Lagrangian density that they started with in [6]. If one starts with a different Lagrangian density, then the result, in more modern notation, will be

$$i\gamma^{\alpha}\nabla_{:\alpha}\psi - \frac{3}{8}l^2(\bar{\psi}\gamma_5\gamma^{\alpha}\psi)\gamma_5\gamma_{\alpha}\psi = m\psi\,, \tag{5}$$

where the colon denotes the covariant derivative and l is Plank length. Which we believe is the proper form for the equation and it also agrees with the action Eq. (2.1) of this paper [10] and Eq. (23) in [12] as far as what the sign should be on the spin-torsion term. However, the sign on the derivative term does not matter because it vanishes for the rest frame analysis we will be concerned with, as in [7]. The second term on the left hand side is the gravitational spin-torsion term. If we include that term in the Lagrangian for quantum electrodynamics we have,

$$\frac{\mathfrak{L}_{\mathrm{QED}}}{\sqrt{-g}} = i\hbar c\,\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + e\,\bar{\psi}\gamma^{\mu}A_{\mu}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - mc^2\bar{\psi}\psi - \frac{3\kappa\hbar^2c^2}{16}(\bar{\psi}\gamma^5\gamma_{\mu}\psi)(\bar{\psi}\gamma^5\gamma^{\mu}\psi). \tag{6}$$

We believe this Lagrangian to be the complete Lagrangian for QED for charged fermions. And if we go to just the rest frame of a charged fermion, we will be only concerned with these terms,

$$\mathfrak{L}_{\mathrm{QED,R.F.}} = +e\,\bar{\psi}\gamma^{0}A_{0}\psi - mc^2\bar{\psi}\psi - \frac{3\kappa\hbar^2c^2}{16}(\bar{\psi}\gamma^5\gamma_{0}\psi)(\bar{\psi}\gamma^5\gamma^{0}\psi), \tag{7}$$

which we use for our derivation in the Appendix. Normal gravity pretty much vanishes in the rest frame of a single elementary fermion such as an electron but the gravitational spin-torsion term doesn't vanish at lengths smaller than the classical radius. We will demonstrate that now for an electron.

If you set eq. (7) to zero, it can be solved as a function of the radius as shown in our semi-classical derivation in the Appendix. The result for an electron is,

$$\frac{\alpha\hbar c}{r} - \frac{3\kappa(\hbar c)^2}{8\,r^3} = m_e c^2, \tag{8}$$

where r is the radius, $\kappa = 8\pi G/c^4$, and $\alpha = e^2/4\pi\hbar c$ is the fine structure constant. It is easy to see that the first term (electrostatic) is balanced by the second term to produce the rest mass-energy of the electron. However, if we use the classical radius, $r = \alpha\hbar c/m_e c^2$ in this equation we will have for the spin-torsion term,

$$\frac{3\kappa(\hbar c)^2}{8\,(\frac{\alpha\hbar c}{m_e c^2})^3} = \frac{3\pi G\hbar^2}{c^2(2.818\times10^{-15}\,\text{meter})^3} \simeq 2.171\times10^{-38}\,\text{MeV}, \tag{9}$$

which is a very small negligible amount of energy produced by the spin-torsion term at the classical radius. Now, if we set the radius to say, $r = 10^{-22}$ meter for which it is thought that an electron must have a radius smaller than that, one obtains,

$$\frac{3\pi G\hbar^2}{c^2(10^{-22}\,\text{meter})^3} \simeq 4.858\times10^{-16}\,\text{MeV}, \tag{10}$$

but still a small amount of energy. And finally if we use Planck length for the radius, we obtain,

$$\frac{3\pi G\hbar^2}{c^2\left(\sqrt{\frac{G\hbar}{c^3}}\right)^3} \simeq 1.151\times10^{20}\,\text{GeV}, \tag{11}$$

a huge amount of energy for an elementary electron! However, it is negative energy relative to the electrostatic field term. The important thing to realize here is that the spin-torsion term field balances out the electrostatic energy and provides a natural cut-off near Planck length so that the self-energy of an electron is not infinite. The radius near Planck length comes out to be $\simeq 5.808\times10^{-34}$ meter. This is pretty amazing that we still obtain the normal rest mass-energy for the electron with this very small "size"! We have prepared a couple of plots, Fig. 1 and Fig. 2, to better see what is happening with the spin-torsion term as a function of length. In Fig. 2 it is easy to see that the spin-torsion term value is negligible as the length goes larger than eq.(10). Possibly a reason why it hasn't been noticed.

For comparison, the energy of the electrostatic term at Planck length is

$$\frac{\alpha\hbar c}{\left(\sqrt{\frac{G\hbar}{c^3}}\right)} \simeq 8.909\times10^{16}\,\text{GeV}. \tag{12}$$

But don't be deceived into thinking this doesn't balance out the spin-torsion term energy because very near to the 5.808×10^{-34} meter solution the electrostatic term and spin-torsion term equal each other. That solution is about 36 times longer than Planck length. When the two terms are equal, we have for r_t

$$r_t = \sqrt{\frac{3\pi}{\alpha}}\;l_P \;\simeq\; 5.8083880810916527435501\,0\times10^{-34}\,\text{meter}, \tag{13}$$

using arbitrary precision in Mathematica [7]. Note that r_t is entirely composed of constants where l_P is Planck length. For comparison, the solution for an electron is

$$r_e \simeq 5.8083880810916527441487\,2\times10^{-34}\,\text{meter}, \tag{14}$$

so very sensitive at this length to 19 significant figures.

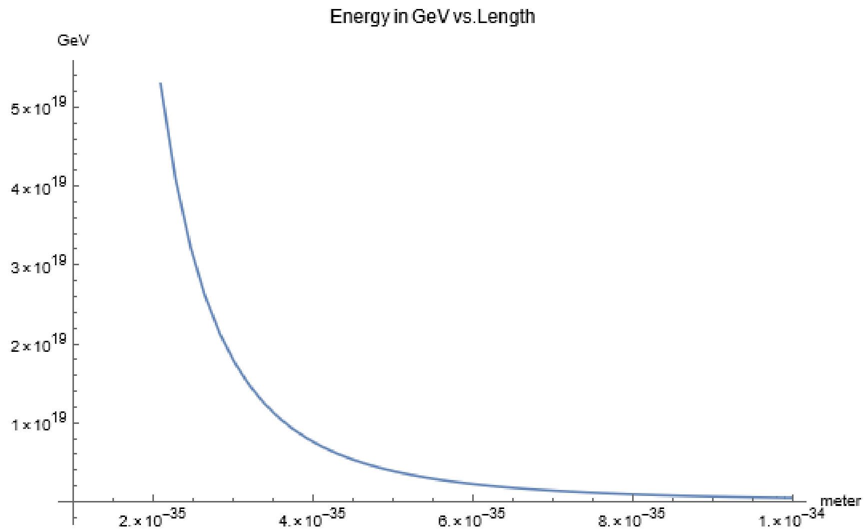

FIG. 1: Energy of the spin-torsion term near Planck length.

Another example where the negative non-linear gravitational spin-torsion term is used is in a paper by Perez and Rovelli [11]. Their Eq. (15) reads:

$$S_{int}[e,\psi] = -\frac{3}{2}\pi G \frac{\gamma^2}{\gamma^2+1}\int d^4x\,\mathbf{e}\,(\bar{\psi}\gamma_5\gamma_A\psi)(\bar{\psi}\gamma_5\gamma^A\psi). \tag{15}$$

The Immirzi parameter, γ, can be taken to infinity so that $\gamma^2/(\gamma^2+1) = 1$, and with $\kappa = 8\pi G$, the standard minimal coupling is recovered:

$$S_{int}[e,\psi] = -\frac{3\kappa}{16}\int d^4x\,\mathbf{e}\,(\bar{\psi}\gamma_5\gamma_A\psi)(\bar{\psi}\gamma_5\gamma^A\psi). \tag{16}$$

In our view, this is the proper form for the action, which makes it clear that the spin-torsion term is negative in the action. The authors seem to have derived this term differently from Hehl and Datta, but, unfortunately, they do not provide the details.

III. COMPARISON WITH STANDARD ELECTRON THEORY

In standard electron theory, the observed mass is supposedly composed of the electromagnetic mass, δm, plus the bare mass, m_0, which would have to be a negative mechanical mass of some sort. We think we have found this negative mechanical mass via the gravitational spin-torsion mechanism. In 1939, Weisskopf calculated the electromagnetic mass and found it to be logarithmically divergent. Ever since then it has been thought that the mechanical "bare" mass would have to be negatively divergent. But the scenario with gravitational spin-torsion added makes that not correct. It turns out that neither the electromagnetic mass nor the "bare" mass are divergent because they are automatically cancelling each other leaving only some electromagnetic mass for the rest mass. In other words, it turns out that the rest mass of an electron is basically entirely electromagnetic as can be seen in our eq.(8).

After a few attempts using standard electron theory with the spin-torsion term, we finally realized it was completely the wrong process to use for electron self-energy. The proper process is to analyze in the rest frame only of the charged fermion. The clue came from our semi-classical analysis which can be found in the appendix of this paper. That means that there is no linear momentum at all and there are no propagators to use from Feynman diagrams.

Upon first appearances, for the rest frame, it seems that the middle term in the non-linear spin-torsion term, $(\bar{\psi}\gamma_k\gamma 5\psi)$, vanishes because $(\bar{\psi}\gamma_0\gamma 5\psi) = 0$ normally, which makes the whole spin-torsion term vanish in the rest frame. By contrast, we have found that fermion anti-fermion mixing is involved in the spin-torsion term in the rest

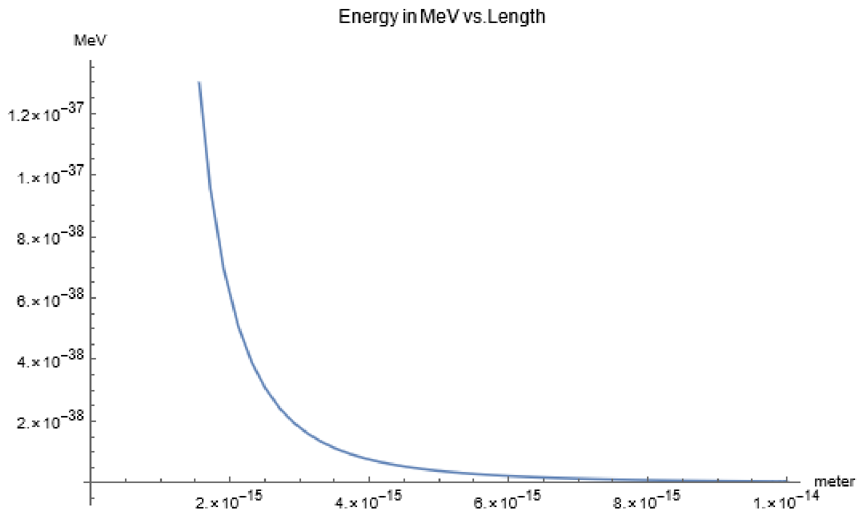

FIG. 2: Energy of the spin-torsion term near the classical radius.

frame of the particle, as we demonstrate below in the Appendix. Moreover, in the full machinery of quantum field theory, ψ can represent a particle or anti-particle in the rest frame, as in the following equations:

$$\psi(0,t) \mid i\,\rangle = \sqrt{\frac{1}{r^3}}\, u^i(m) e^{-iEt} \quad \text{or} \quad \psi(0,t) \mid i\,\rangle = \sqrt{\frac{1}{r^3}}\, v^i(m) e^{+iEt}, \tag{17}$$

where the $\mid i\,\rangle$ represents the initial state with u^i for the particle and v^i the anti-particle. In order for the spin-torsion term to be non-zero in the rest frame, one must have one of the ψ's represent an appropriate anti-fermion, as in the following equations:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \gamma^5 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 1 \quad \text{or} \quad \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} \gamma^5 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 1. \tag{18}$$

This fermion anti-fermion mixing is a bit of a mystery but highly probable by interaction with the quantum vacuum. However, the mixing is highly necessary or the electron would blow itself up with infinite energy if the spin-torsion term were zero.

IV. FURTHER DISCUSSION AND QUESTIONS

So, we have a Planck "sized" electron that is considered an electro-weak particle energy-wise. And why two different radii? We have the rest mass-energy at the classical radius and also at the radius near Planck length. Why? Fig. 3 shows the behavior of the function, $E(r)$, of the electrostatic energy of an electron minus the spin-torsion energy as a function of length which explains some of this,

$$E(r) = \frac{\alpha\hbar c}{r} - \frac{3\kappa(\hbar c)^2}{8\,r^3}. \tag{19}$$

The solution for $m_e c^2$ near Planck length is on the very steep slope on the left hand side of the plot whereas the solution at the classical radius is way off to the right hand side of the figure. And notice that we have a peak energy of about 9.54173×10^{14} GeV at length of about 1.006×10^{-33} meters. This is reminiscent of GUT scale energies. Could there be charged fermions with this much energy? And what locks the electron's energy to $m_e c^2$? So, we have more mysteries to investigate. This scenario also applies to muons and tauons except they aren't stable particles. For quarks, $E(r)$ becomes much more complicated since you have color charge added to electromagnetic charge. Then we

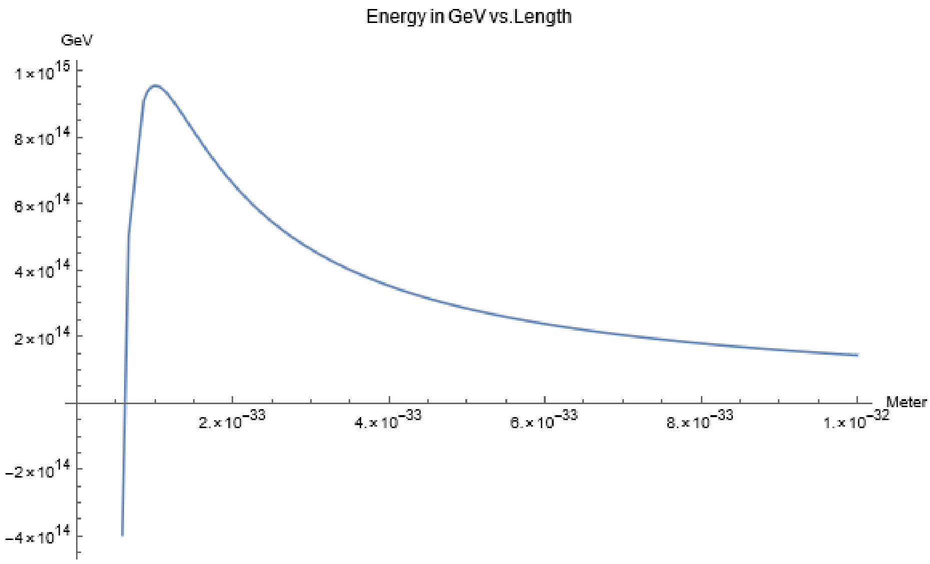

FIG. 3: $E(r) = \alpha\hbar c/r - 3\kappa(\hbar c)^2/8\,r^3$ near Planck length.

have to suppose for neutrinos, that it would be weak charge in formula for $E(r)$. Just replace α for electromagnetics with α_w for weak charge.

We suspect that the classical radius is primarily due to screening of the charge by virtual particles from the quantum vacuum. If it were possible to see near Planck length via scattering of some kind after breaking through the screening effect, one would see the radius near Planck length. The screening effect of interaction with the quantum vacuum "spreads" out the apparent charge of an electron so that one might think there is a "bare" charge if the screening wasn't there [13]. Zitterbewegung is the clue for this. The point-like entity of the electron near Planck length vibrates around in the quantum vacuum interacting with virtual electrons and positrons with an amplitude of about the classical radius.

Let's now investigate the "standard" result for the logarithmically divergent electron self-energy which is the orthodox solution for electromagnetic mass, δm, [13] so,

$$\delta m = \frac{3m\alpha}{2\pi}\log\frac{\Lambda}{m}, \tag{20}$$

where Λ is an arbitrary cutoff when taken to infinity gives the logarithmically divergent result. However, the m in that expression enters via a propagator when using a quantum field theory S-matrix evaluation thus is virtual and can be any value. That allows us to make a substitution $m = 1/r$ so that,

$$\delta m = \frac{3\alpha}{2\pi r}\log\frac{r}{\Lambda}, \tag{21}$$

which is an expression like that for electrostatic energy and probably more familiar and Λ is now a length instead mass. Of course this expression is still logarithmically divergent as $\Lambda \to 0$ or $r \to 0$. If we now set $r = \Lambda$, we get zero for electromagnetic mass so that makes us think the expression is nonsense. In fact, if $m = \Lambda$ in the first equation we also get zero which is possible since the mass is virtual. That is the main reason why our attempts to use the "standard" result failed. It is in fact infinite nonsense! Milonni shows the observed mass as [13]

$$m_{obs} = m_0 + \delta m = m_0 + \frac{3m_0\alpha}{2\pi}\log\frac{\Lambda}{m_0}, \tag{22}$$

where m_0 is the "bare" mass. Since m_0 is in the δm result, that reinforces our notion of doing $m = 1/r$. Also,the observed mass is dependent on an arbitrary cutoff. The argument to fix that is that the "bare" mass also depends on the cutoff. That seems fairly convoluted. With our situation things are simple; you just subtract the "bare" mass (spin-torsion term) from the electrostatic mass to obtain the observed rest frame mass.

It is pretty amazing that the fairly simple spin-torsion term completes electrodynamics. No more renormalization is required for many calculations although it could be handy for some. So, we see with the spin-torsion term scenario there are still plenty of questions and mysteries of Nature to solve.

Acknowledgement

The authors would like to thank Luca Fabbri for many discussions about gravitational torsion and letting us know that we are on a good track.

Appendix: Semi-Classical Derivation of Charged Fermion Self-Energy

In the rest frame, where normal gravity is effectively zero for an elementary fermion, with natural units $\hbar = c = 1$, along with the electrostatic term via a local gauge transformation, we have from the Lagrangian eq.(7),

$$i\,\gamma^0 \frac{\partial \psi}{\partial t} + qA_0\,\gamma^0 \psi = m\,\psi + \frac{3\kappa}{8}\left(\bar{\psi}\gamma^5\gamma_0\psi\right)\gamma^5\gamma^0\psi\,, \tag{A.1}$$

which can be further simplified to

$$i\begin{pmatrix} +\frac{\partial\psi_1}{\partial t} \\ +\frac{\partial\psi_2}{\partial t} \\ -\frac{\partial\psi_3}{\partial t} \\ -\frac{\partial\psi_4}{\partial t} \end{pmatrix} + qA_0 \begin{pmatrix} +\psi_1 \\ +\psi_2 \\ -\psi_3 \\ -\psi_4 \end{pmatrix} = m \begin{pmatrix} +\psi_1 \\ +\psi_2 \\ +\psi_3 \\ +\psi_4 \end{pmatrix} - \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\} \begin{pmatrix} -\psi_3 \\ -\psi_4 \\ +\psi_1 \\ +\psi_2 \end{pmatrix}, \tag{A.2}$$

where we have used

$$\gamma^0 = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \text{and} \qquad \gamma^5 = \begin{pmatrix} 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \end{pmatrix}. \tag{A.3}$$

If we now represent the particles and anti-particles with two-component spinors ψ_a and ψ_b, respectively [18], where

$$\psi_a := \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad \text{and} \quad \psi_b := \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} \tag{A.4}$$

are the two-component spinors constituting the four-component Dirac spinor, then the above equation (A.1) can be written as two *coupled* partial differential equations:

$$+\,i\,\frac{\partial\psi_a}{\partial t} + qA_0\,\psi_a = m\,\psi_a + \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\}\,\psi_b \tag{A.5}$$

$$-\,i\,\frac{\partial\psi_b}{\partial t} - qA_0\,\psi_b = m\,\psi_b - \frac{3\kappa}{8}\left\{\psi_1^*\psi_3 + \psi_2^*\psi_4 + \psi_1\psi_3^* + \psi_2\psi_4^*\right\}\,\psi_a\,. \tag{A.6}$$

It is now very easy to see the fermion anti-fermion mixing in the spin-torsion non-linear term, which can be seen also in Eq. (A.2).

Unlike the case in Dirac equation, these equations for the spinors ψ_a and ψ_b are coupled equations even in the rest frame. They decouple in the limit when the torsion-induced axial-axial self-interaction is negligible. On the other hand, at low energies it is reasonable to assume that, in analogy with the Dirac spinors in flat spacetime, the above two-component spinors for free particles decouple in the rest frame, admitting plane wave solutions of the form

$$\psi_a(t) = \sqrt{\frac{1}{V}}\,e^{-iEt}\psi_a(0) \quad \text{and} \quad \psi_b(t) = \sqrt{\frac{1}{V}}\,e^{+iEt}\psi_b(0), \tag{A.7}$$

where $E = m = \omega = 2\pi/t$ in the rest frame. We now note that in the rest frame the derivative term in the eqs. (A.5) and (A.6) vanishes since ψ is constant and the kinetic energy in the rest frame would be zeo anyways, also because in the rest frame the probability of finding the particle in a given volume at time t is one. Moreover $E = 2\pi/t$ gives

$$\psi_a(t) = \psi_b(t). \tag{A.8}$$

Consequently eqs. (A.5) and (A.6) uncouple and simplify to

$$+qA_0\,\psi_a = m\,\psi_a + \frac{3\kappa}{8}\left\{\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3\psi_3^* + \psi_4\psi_4^*\right\}\psi_a\,, \tag{A.9}$$

$$-qA_0\,\psi_b = m\,\psi_b - \frac{3\kappa}{8}\left\{\psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_3\psi_3^* + \psi_4\psi_4^*\right\}\psi_b\,. \tag{A.10}$$

Substituting the ψ's from eq. (A.7) and simplifying reduces eqs. (A.9) and (A.10) to the following pair of equations:

$$+qA_0\psi_a(0) - \frac{3\kappa}{8\,r^3}\,|\psi(0)|^2\,\psi_a(0) = m\,\psi_a(0) \tag{A.11}$$

$$\text{and} \quad -\,qA_0\psi_b(0) + \frac{3\kappa}{8\,r^3}\,|\psi(0)|^2\,\psi_b(0) = m\,\psi_b(0)\,. \tag{A.12}$$

And since $|\psi(0)|^2 = 1$ in the rest frame, these equations can be further simplified to

$$+\,qA_0 - \frac{3\kappa}{8\,r^3} = m\,, \tag{A.13}$$

$$\text{and} \quad -\,qA_0 + \frac{3\kappa}{8\,r^3} = m\,. \tag{A.14}$$

Substituting in natural units for the scalar field $A_0 = V = q/(4\pi r)$ in the Lorentz gauge (where V is the electric potential), and for $\kappa = 8\pi G$, we finally arrive at our central equations, for any electroweak fermion of charge q and mass m and its anti-particle in the Riemann-Cartan spacetime:

$$+\frac{q^2}{4\pi r} - \frac{3\pi G}{r^3} = +m \tag{A.15}$$

$$\text{and} \quad -\frac{q^2}{4\pi r} + \frac{3\pi G}{r^3} = +m, \tag{A.16}$$

where r is the radial distance from q and the two equations correspond to the particle and anti-particle, respectively. Finally, replacing q with electron or positron charge and replacing $\hbar$ and c gives us our central equation for a fermion for our semi-classical evaluation:

$$\frac{\alpha\hbar c}{r} - \frac{3\kappa(\hbar c)^2}{8\,r^3} = mc^2. \tag{A.17}$$

This same equation is also derived via a S-matrix quantum field theory evaluation for the rest frame in [7].

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