Local Quantum Mechanical Prediction of the Singlet State

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The following is a way of deriving the quantum mechanical (QM) prediction for the EPR-Bohm experiment (EPRB) [1] via local measurement functions [2]. We have given in the Appendix a standard QM derivation for comparison. Consider a pair of spin one-half particles, moving freely after production in opposite directions, with particles 1 and 2 subject, respectively, to spin measurements along independently chosen unit directions $\mathbf{a}$ and $\mathbf{b}$, which can be located at a spacelike distance from each other. If initially the pair has vanishing total spin, then the pair's quantum mechanical spin state would be the following entangled singlet state:

$$|\Psi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1 \otimes |\mathbf{n},\,-\rangle_2 \,-\, |\mathbf{n},\,-\rangle_1 \otimes |\mathbf{n},\,+\rangle_2\Big\}, \tag{1}$$

and,

$$\boldsymbol{\sigma}\cdot\mathbf{n}\,|\mathbf{n},\,\pm\rangle \,=\, \pm\,|\mathbf{n},\,\pm\rangle, \tag{2}$$

describing the quantum mechanical eigenstates in which the particles have spin "up" or "down" in units of $\hbar = 2$, with $\boldsymbol{\sigma}$ being the familiar Pauli spin "vector" $(\sigma_x,\,\sigma_y,\,\sigma_z)$.

Quantum mechanically the rotational invariance of the singlet state $|\Psi_{\mathbf{n}}\rangle$ ensures that the expectation values of the individual spin observables $\boldsymbol{\sigma}_1\cdot\mathbf{a}$ and $\boldsymbol{\sigma}_2\cdot\mathbf{b}$ are

$$\begin{aligned} \mathcal{E}_{q.m.}(\mathbf{a}) \,&=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\mathbb{1}\,|\Psi_{\mathbf{n}}\rangle \,=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\,|\Psi_{\mathbf{n}}\rangle \,=\, 0 \\ \text{and}\quad \mathcal{E}_{q.m.}(\mathbf{b}) \,&=\, \langle\Psi_{\mathbf{n}}|\,\mathbb{1}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \,=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \,=\, 0\,, \end{aligned} \tag{3}$$

where $\mathbb{1}$ is the identity matrix. The expectation value of the joint observable $\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}$ is

$$\mathcal{E}_{q.m.}(\mathbf{a},\,\mathbf{b}) \,=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\,\otimes\,\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \,=\, -\,\mathbf{a}\cdot\mathbf{b}\,, \tag{4}$$

regardless of the relative distance between the two remote locations represented by the unit vectors $\mathbf{a}$ and $\mathbf{b}$.

We will now construct some manifestly local measurement functions that derives the above result of $-\mathbf{a}\cdot\mathbf{b}$ and agrees with the eigenvalues of the observable operators which involve spins being detected by detectors with the single vector:

$$\mathbf{s} \,=\mathbf{s}_1 = -\mathbf{s}_2 \tag{5}$$

$$\mathbf{r}_a \,=\mathbf{a}\times\mathbf{s}_1 \;\text{ and }\; \mathbf{r}_b = \mathbf{s}_2\times\mathbf{b} \tag{6}$$

$$\begin{aligned} A(\mathbf{a},\,\mathbf{s_1}) :=& \lim_{\mathbf{s_1}\to\,\mathrm{sgn}(\mathbf{a}\cdot\mathbf{s_1})\mathbf{a}}\Big[\langle\phi_{\mathbf{n}}|\{(\boldsymbol{\sigma}\cdot\mathbf{a})\,(\boldsymbol{\sigma}\cdot\mathbf{s_1})\}|\phi_{\mathbf{n}}\rangle + \mathbf{r}_a\,\sin\left(\eta_{as_1}\right)\Big] \\ =& \lim_{\mathbf{s_1}\to\,\mathrm{sgn}(\mathbf{a}\cdot\mathbf{s_1})\mathbf{a}}\Big[\mathbf{q}(\eta_{\mathbf{a}\mathbf{s}_1},\,\mathbf{r}_a)\Big] = \mathrm{sgn}(\mathbf{a}\cdot\mathbf{s_1}) = \pm 1 \end{aligned} \tag{7}$$

$$\begin{aligned} \text{and}\quad B(\mathbf{b},\,\mathbf{s_2}) :=& \lim_{\mathbf{s_2}\to\,\mathrm{sgn}(\mathbf{b}\cdot\mathbf{s_2})\mathbf{b}}\Big[\langle\chi_{\mathbf{n}}|\{(\boldsymbol{\sigma}\cdot\mathbf{s_2})\,(\boldsymbol{\sigma}\cdot\mathbf{b})\}|\chi_{\mathbf{n}}\rangle + \mathbf{r}_b\,\sin\left(\eta_{s_1b}\right)\Big] \\ =& \lim_{\mathbf{s_2}\to\,\mathrm{sgn}(\mathbf{b}\cdot\mathbf{s_2})\mathbf{b}}\Big[\mathbf{q}(\eta_{\mathbf{s}_1\mathbf{b}},\,\mathbf{r}_b)\Big] = \mathrm{sgn}(\mathbf{s_2}\cdot\mathbf{b}) = \pm 1, \end{aligned} \tag{8}$$

where

$$|\phi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1|\mathbf{n},\,-\rangle_3\Big\} \tag{9}$$

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$$\text{and} \quad |\chi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_4|\mathbf{n},\,-\rangle_2\Big\}. \tag{10}$$

Here $\boldsymbol{\sigma}\cdot\mathbf{a}$ and $\boldsymbol{\sigma}\cdot\mathbf{b}$ represent the detectors of Alice and Bob with no angular momentum at time of detection, and $\boldsymbol{\sigma}\cdot\mathbf{s_1} = -\boldsymbol{\sigma}\cdot\mathbf{s_2}$ represents the spin of the fermions they receive, for which the EPRB experiment is being performed. The replacement limit functions express the action of the polarizers at the detection stations and that $|\phi_{\mathbf{n}}\rangle$ and $|\chi_{\mathbf{n}}\rangle$ are simple products and now represent the wavefunction of the separate particles. The original singlet is now split between two different simple product bra-kets. And we see in the second step of the A and B functions that we have quaternions defined from the first step.

Note that the measurement functions represent simultaneous detection processes occurring at two possibly spacelike separated observation stations of Alice and Bob. Although occurring simultaneously, $A(\mathbf{a},\,\mathbf{s_1})$ and $B(\mathbf{b},\,\mathbf{s_2})$ are independent physical processes that are *not* subject to the conservation of the initial zero spin angular momentum. Before proceeding with the product calculation, we will implement a notation simplification, $\mu_{\mathbf{n}} = \text{sgn}(\mathbf{n}\cdot\mathbf{s})\mathbf{n}$, plus the k indices will be suppressed after the first step. Upon using the "product of limits equal to limits of product" rule, leads to the expectation value calculated as follows [5]:

$$\mathcal{E}(\mathbf{a},\,\mathbf{b}) \;=\; \lim_{n>>1}\left[\frac{1}{n}\sum_{k=1}^{n} A(\mathbf{a}^k,\,\mathbf{s}_1^k)\,B(\mathbf{b}^k,\,\mathbf{s}_2^k)\right] \tag{11}$$

$$= \lim_{n\gg 1}\left\{\frac{1}{n}\sum_{k=1}^{n}\left[\lim_{\mathbf{s}_1\to\mu_a}\{\,\mathbf{q}(\eta_{\mathbf{as}_1},\,\mathbf{r}_a)\}\right]\left[\lim_{\mathbf{s}_2\to\mu_b}\{\,\mathbf{q}(\eta_{\mathbf{s}_2\mathbf{b}},\,\mathbf{r}_b)\}\right]\right\} \tag{12}$$

$$= \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{\mathbf{s}_1\to\mu_a\\ \mathbf{s}_2\to\mu_b}}\Big\{\mathbf{q}(\eta_{\mathbf{as}_1},\,\mathbf{r}_a)\;\mathbf{q}(\eta_{\mathbf{s}_2\mathbf{b}},\,\mathbf{r}_b)\Big\}\right] \tag{13}$$

$$= \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{\mathbf{s}_1\to\mu_a\\ \mathbf{s}_2\to\mu_b}}\Big\{[\cos(\,\eta_{\mathbf{as}_1}) + \mathbf{r}_a\,\sin(\,\eta_{\mathbf{as}_1})]\,[\cos(\,\eta_{\mathbf{s}_2\mathbf{b}}) + \mathbf{r}_b\,\sin(\,\eta_{\mathbf{s}_2\mathbf{b}})]\Big\}\right] \tag{14}$$

$$= \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{\mathbf{s}_1\to\mu_a\\ \mathbf{s}_2\to\mu_b}}\Big\{-\mathbf{q}(\eta_{\mathbf{ab}},\,\mathbf{r}_0)\Big\}\right] \tag{15}$$

$$= \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\lim_{\substack{\mathbf{s}_1\to\mu_a\\ \mathbf{s}_2\to\mu_b}}\Big\{-\cos(\,\eta_{\mathbf{ab}}) - \mathbf{L}\,(\mathbf{r}_0)\sin(\,\eta_{\mathbf{ab}})\Big\}\right] \tag{16}$$

$$= \;-\cos(\,\eta_{\mathbf{ab}}) - \lim_{n\gg 1}\left[\frac{1}{n}\sum_{k=1}^{n}\mathbf{L}\left(\vec{\mathbf{0}}\right)\,\sin(\,\eta_{\mathbf{ab}})\right] \tag{17}$$

$$= -\cos(\,\eta_{\mathbf{ab}}) = -\mathbf{a}\cdot\mathbf{b}, \tag{18}$$

where in step (14) we have used the cosine and sine representation of the quaternions. When that is expanded and calculated out you get step (15). Thus we obtain the correct result via a completely local process. So, we can see here that QM is completed by 3-sphere topology. And that the singlet has a parallelized 3-sphere topology [4, 5]. This is very easy to demonstrate via the Pauli algebra. We have validated the above analytical calculation via a computer simulation using the programming language of the mathematical program Mathematica which is presented in Appendix B.

Appendix A: Standard QM Derivation

The expectation value of the joint observable $\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}$ is

$$\mathcal{E}_{q.m.}(\mathbf{a},\,\mathbf{b}) \;=\; \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\;\otimes\;\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \;=\; -\,\mathbf{a}\cdot\mathbf{b}\,, \tag{A1}$$

regardless of the relative distance between the two remote locations represented by the unit vectors $\mathbf{a}$ and $\mathbf{b}$. This result can be derived using the following calculation for the singlet state [3],

$$\mathcal{E}_{q.m.}(\mathbf{a},\,\mathbf{b}) \;=\; \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}\cdot\mathbf{a}\;\otimes\;\boldsymbol{\sigma}\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \tag{A2}$$

$$= \frac{1}{2}\Big[\begin{pmatrix}1 & 0\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{a}) \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0 & 1\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{b}) \begin{pmatrix}0\\1\end{pmatrix} - \begin{pmatrix}0 & 1\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{a}) \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}1 & 0\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{b}) \begin{pmatrix}0\\1\end{pmatrix}$$
$$- \begin{pmatrix}1 & 0\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{a}) \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}0 & 1\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{b}) \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0 & 1\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{a}) \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}1 & 0\end{pmatrix} (\boldsymbol{\sigma}\cdot\mathbf{b}) \begin{pmatrix}1\\0\end{pmatrix}\Big], \tag{A3}$$

$$= \frac{1}{2}\Big[\begin{pmatrix}1 & 0\end{pmatrix} \begin{pmatrix} a_z & a_x - ia_y \\ a_x + ia_y & -a_z \end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}0 & 1\end{pmatrix} \begin{pmatrix} b_z & b_x - ib_y \\ b_x + ib_y & -b_z \end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix}$$
$$- \begin{pmatrix}0 & 1\end{pmatrix} \begin{pmatrix} a_z & a_x - ia_y \\ a_x + ia_y & -a_z \end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix} \begin{pmatrix}1 & 0\end{pmatrix} \begin{pmatrix} b_z & b_x - ib_y \\ b_x + ib_y & -b_z \end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix}$$
$$- \begin{pmatrix}1 & 0\end{pmatrix} \begin{pmatrix} a_z & a_x - ia_y \\ a_x + ia_y & -a_z \end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}0 & 1\end{pmatrix} \begin{pmatrix} b_z & b_x - ib_y \\ b_x + ib_y & -b_z \end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix}$$
$$+ \begin{pmatrix}0 & 1\end{pmatrix} \begin{pmatrix} a_z & a_x - ia_y \\ a_x + ia_y & -a_z \end{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \begin{pmatrix}1 & 0\end{pmatrix} \begin{pmatrix} b_z & b_x - ib_y \\ b_x + ib_y & -b_z \end{pmatrix} \begin{pmatrix}1\\0\end{pmatrix}\Big], \tag{A4}$$

$$= \frac{1}{2}\{(-a_z b_z) + (-a_x b_x - a_y b_y) + (-a_x b_x - a_y b_y) + (-a_z b_z)\}, \tag{A5}$$

$$= -\,\mathbf{a}\cdot\mathbf{b}\,. \tag{A6}$$

One might wonder why this derivation gives the same result as the previous derivation with no $(\boldsymbol{\sigma}\cdot\mathbf{s_n})$ particle spin representations. In this derivation, the particles are represented by the up and down probabilities from the $\Psi_\mathbf{n}$ wavefunction.

Appendix B: Simulation Code for Verification of the Singlet State Local QM Prediction

We present here the simulation code for the validation of the local QM prediction of $-\mathbf{a}\cdot\mathbf{b}$ using the programming language of the mathematical program Mathematica. The Mathematica notebook file is available at [6] see also [7].

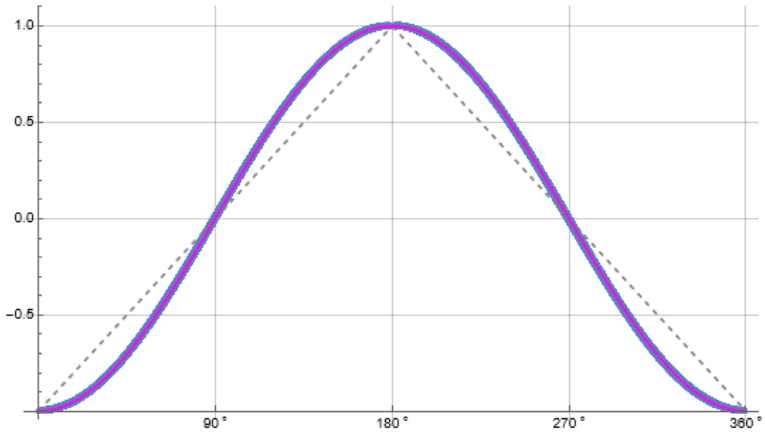
```
<< Quaternions`
β0 = Quaternion[1, 0, 0, 0];
β1 = Quaternion[0, 1, 0, 0];
β2 = Quaternion[0, 0, 1, 0];
β3 = Quaternion[0, 0, 0, 1];
Qcoordinates = {β0, β1, β2, β3};
m = 20 000;
s1 = ConstantArray[0, m];
s2 = ConstantArray[0, m];
σs1 = ConstantArray[0, m];
σs2 = ConstantArray[0, m];
a1 = ConstantArray[0, m];
b1 = ConstantArray[0, m];
ra1 = ConstantArray[0, m];
rb1 = ConstantArray[0, m];
qA = ConstantArray[0, m];
qB = ConstantArray[0, m];
A = ConstantArray[0, m];
B = ConstantArray[0, m];
pc=ConstantArray[0, m];
plotAB = Table[{0, 0}, m];

Do[s = RandomPoint[Sphere[ ]]; (*Uniform Unit 3D Vectors; Hidden Variable*)
s1[[i]] = s;
s2[[i]] = −s;
σs1[[i]] = PauliMatrix[1] ∗ s[[1]] + PauliMatrix[2] ∗ s[[2]] + PauliMatrix[3] ∗ s[[3]];
σs2[[i]] = −(PauliMatrix[1] ∗ s[[1]] + PauliMatrix[2] ∗ s[[2]] + PauliMatrix[3] ∗ s[[3]]), {i, m}]

Do[a = RandomPoint[Sphere[ ]]; (*Uniform Unit 3D Vectors*)
```

a1[[i]] = a;
σa = PauliMatrix[1] $*$ a[[1]] + PauliMatrix[2] $*$ a[[2]] + PauliMatrix[3] $*$ a[[3]];
cosas1 = Extract $\left[\text{Re}\left[\text{Flatten}\left[\frac{1}{2}\left(\begin{pmatrix}1 & 0\end{pmatrix}.\sigma\text{a}.\sigma\text{s1}[[i]].\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}.\sigma\text{a}.\sigma\text{s1}[[i]].\begin{pmatrix}0\\1\end{pmatrix}\right)\right]\right],1\right]$;
(*Particle - Detector interaction*)
ra = Cross[a, s1[[i]]];
ra1[[i]] = ra;
qA[[i]] = {cosas1, ra[[1]], ra[[2]], ra[[3]]}.Qcoordinates;
A[[i]] = Sign[cosas1], {i, m}]

Do[b = RandomPoint[Sphere[]]; (*Uniform Unit 3D Vectors*)
b1[[i]] = b;
σb = PauliMatrix[1] $*$ b[[1]] + PauliMatrix[2] $*$ b[[2]] + PauliMatrix[3] $*$ b[[3]];
cosbs2 = Extract $\left[\text{Re}\left[\text{Flatten}\left[\frac{1}{2}\left(\begin{pmatrix}1 & 0\end{pmatrix}.\sigma\text{s2}[[i]].\sigma\text{b}.\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0 & 1\end{pmatrix}.\sigma\text{s2}[[i]].\sigma\text{b}.\begin{pmatrix}0\\1\end{pmatrix}\right)\right]\right],1\right]$;
(*Particle - Detector interaction*)
rb = Cross[s2[[i]], b];
rb1[[i]] = rb;
qB[[i]] = {cosbs2, rb[[1]], rb[[2]], rb[[3]]}.Qcoordinates;
B[[i]] = Sign[cosbs2], {i, m}]

Do[
r0 =
(Re[qA[[i]]]Limit[Cross[s22, b1[[i]]], s22 $\to$ Sign[Re[qA[[i]]]]b1[[i]]]+
Re[qB[[i]]]Limit[Cross[a1[[i]], s11], s11 $\to$ Sign[Re[qA[[i]]]]a1[[i]]]$-$
Cross[Limit[Cross[a1[[i]], s11], s11 $\to$ Sign[Re[qA[[i]]]]a1[[i]]],
Limit[Cross[s22, b1[[i]]], s22 $\to$ Sign[Re[qB[[i]]]]b1[[i]]]])/
(Sin[ArcCos[a1[[i]].b1[[i]]]]);
qpc = {(Re[qA[[i]]] $*$ Re[qB[[i]]] $-$ ra1[[i]].rb1[[i]]), r0[[1]], r0[[2]], r0[[3]]}.Qcoordinates;
(*Product Calculation*)
pc[[i]] = FromQuaternion[qpc];
ϕa = ArcTan[a1[[i]][[1]], a1[[i]][[2]]];
ϕb = ArcTan[b1[[i]][[2]], b1[[i]][[1]]];
If[ϕa $*$ ϕb $>$ 0, angle = ArcCos[a1[[i]].b1[[i]]]/Degree,
angle = (2π $-$ ArcCos[a1[[i]].b1[[i]]])/Degree];
plotAB[[i]] = {angle, qpc[[1]]}, {i, m}]

simulation = ListPlot[plotAB, PlotMarkers $\to$ {Automatic, Small}, AspectRatio $\to$ 9/16,
Ticks $\to$ {{{90, 90°}, {180, 180°}, {0, 0°}, {270, 270°}, {360, 360°}}, Automatic},
GridLines $\to$ Automatic, AxesOrigin $\to$ {0, $-$1.0}];
negcos = Plot[$-$Cos[xDegree], {x, 0, 360}, PlotStyle $\to$ {Magenta}];
p1 = Plot[$-$1 + 2x1Degree/π, {x1, 0, 180}, PlotStyle $\to$ {Gray, Dashed}];
p2 = Plot[3 $-$ 2x2Degree/π, {x2, 180, 360}, PlotStyle $\to$ {Gray, Dashed}];
Show[simulation, p1, p2, negcos]



Blue is the correlation data, magenta is the negative cosine curve for an exact match.

AveA = N[Total[A]/m];
AveB = N[Total[B]/m];
Print[" <A> = ", AveA, " <B> = ", AveB];
meanpc = Mean[pc];
Print["Imaginary part vanishes, meanpc = ", meanpc];

<A> = -0.0082 <B> = -0.0001

Imaginary part vanishes, meanpc = $-0.00511306 + 0.i$

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