Local Quantum Mechanical Prediction of the Singlet State

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(Dated: 3-24-2022)

We derive the quantum mechanical prediction of $-\mathbf{a}\cdot\mathbf{b}$ for the singlet spin state using local measurement functions in the manner of Bell, and verify our derivation in a computer simulation using the programming language of the program Mathematica.

In this letter we derive the quantum mechanical (QM) prediction for the EPR-Bohm experiment (EPRB) [1] using the singlet spin state via local measurement functions in the manner of Bell [2][3]. Consider a pair of spin one-half particles, moving freely after production in opposite directions, with particles 1 and 2 subject, respectively, to spin measurements along independently chosen unit directions $\mathbf{a}$ and $\mathbf{b}$, which can be located at a spacelike distance from each other. If initially the pair has vanishing total spin, then the pair's quantum mechanical spin state would be the following entangled singlet state:

$$|\Psi_{\mathbf{n}}\rangle = \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1\otimes|\mathbf{n},\,-\rangle_2 - |\mathbf{n},\,-\rangle_1\otimes|\mathbf{n},\,+\rangle_2\Big\}, \quad (1)$$

where

$$\boldsymbol{\sigma}\cdot\mathbf{n}\,|\mathbf{n},\,\pm\rangle \,=\, \pm\,|\mathbf{n},\,\pm\rangle, \quad (2)$$

describs the quantum mechanical eigenstates in which the particles have spin "up" or "down" in units of $\hbar = 2$, with $\boldsymbol{\sigma}$ being the familiar Pauli spin "vector" $(\sigma_x,\,\sigma_y,\,\sigma_z)$.

Quantum mechanically the rotational invariance of the singlet state $|\Psi_{\mathbf{n}}\rangle$ ensures that the expectation values of the individual spin observables $\boldsymbol{\sigma}_1\cdot\mathbf{a}$ and $\boldsymbol{\sigma}_2\cdot\mathbf{b}$ are

$$\begin{aligned}
\mathcal{E}_{q.m.}(\mathbf{a}) \,&=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\mathbb{1}\,|\Psi_{\mathbf{n}}\rangle \\
&=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\,|\Psi_{\mathbf{n}}\rangle \,=\, 0 \quad (3)\\
\text{and}\quad \mathcal{E}_{q.m.}(\mathbf{b}) \,&=\, \langle\Psi_{\mathbf{n}}|\,\mathbb{1}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \\
&=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \,=\, 0\,, \quad (4)
\end{aligned}$$

where $\mathbb{1}$ is the identity matrix. The expectation value of the joint observable $\boldsymbol{\sigma}_1\cdot\mathbf{a}\otimes\boldsymbol{\sigma}_2\cdot\mathbf{b}$ is [4]

$$\mathcal{E}_{q.m.}(\mathbf{a},\,\mathbf{b}) \,=\, \langle\Psi_{\mathbf{n}}|\,\boldsymbol{\sigma}_1\cdot\mathbf{a}\,\otimes\,\boldsymbol{\sigma}_2\cdot\mathbf{b}\,|\Psi_{\mathbf{n}}\rangle \,=\, -\,\mathbf{a}\cdot\mathbf{b}\,, \quad (5)$$

regardless of the relative distance between the two remote locations represented by the unit vectors $\mathbf{a}$ and $\mathbf{b}$.

We will now construct some manifestly local measurement functions, in the manner of Bell [2], which leads to the above result of $-\mathbf{a}\cdot\mathbf{b}$ and agrees with the eigenvalues of the observable operators which involve spins being detected by detectors with the single vector split dictated by the conservation of spin angular momentum:

$$\mathbf{s} \,=\mathbf{s}_1 = -\mathbf{s}_2. \quad (6)$$

Using this, we define

$$\begin{aligned}
\mathbf{r}_a \,&= \mathbf{a}\times\mathbf{s}_1 \;\text{ and }\; \mathbf{r}_b = \mathbf{s}_2\times\mathbf{b} \quad (7)\\
\mu_{\mathbf{n}} &= \operatorname{sgn}(\mathbf{n}\cdot\mathbf{s}_x)\mathbf{n} \quad (8)\\
A(\mathbf{a},\,\mathbf{s_1}) &:= \lim_{\mathbf{s_1}\to\,\mu_a}\Big[\langle\phi_{\mathbf{n}}|(\boldsymbol{\sigma}\cdot\mathbf{a})\,(\boldsymbol{\sigma}\cdot\mathbf{s_1})|\phi_{\mathbf{n}}\rangle + \mathbf{r}_a\,\sin\left(\eta_{\mathbf{as_1}}\right)\Big] \\
&= \lim_{\mathbf{s_1}\to\,\mu_a}\Big[\mathbf{q}(\eta_{\mathbf{as_1}},\,\mathbf{r}_a)\Big] \\
&= \operatorname{sgn}(\mathbf{a}\cdot\mathbf{s_1}) = \pm 1 \quad (9)\\
B(\mathbf{b},\,\mathbf{s_2}) &:= \lim_{\mathbf{s_2}\to\,\mu_b}\Big[\langle\chi_{\mathbf{n}}|(\boldsymbol{\sigma}\cdot\mathbf{s_2})\,(\boldsymbol{\sigma}\cdot\mathbf{b})|\chi_{\mathbf{n}}\rangle + \mathbf{r}_b\,\sin\left(\eta_{\mathbf{s_1b}}\right)\Big] \\
&= \lim_{\mathbf{s_2}\to\,\mu_b}\Big[\mathbf{q}(\eta_{\mathbf{s_1b}},\,\mathbf{r}_b)\Big] \\
&= \operatorname{sgn}(\mathbf{s_2}\cdot\mathbf{b}) = \pm 1, \quad (10)
\end{aligned}$$

where

$$\begin{aligned}
|\phi_{\mathbf{n}}\rangle &= \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_1|\mathbf{n},\,-\rangle_3\Big\} \quad (11)\\
\text{and}\quad |\chi_{\mathbf{n}}\rangle &= \frac{1}{\sqrt{2}}\Big\{|\mathbf{n},\,+\rangle_4|\mathbf{n},\,-\rangle_2\Big\}. \quad (12)
\end{aligned}$$

Here $\boldsymbol{\sigma}\cdot\mathbf{a}$ and $\boldsymbol{\sigma}\cdot\mathbf{b}$ represent the detectors of Alice and Bob with no angular momentum at time of detection, and $\boldsymbol{\sigma}\cdot\mathbf{s_1} = -\boldsymbol{\sigma}\cdot\mathbf{s_2}$ represents the spin of the fermions they receive, for which the EPRB experiment is being performed. The replacement limit functions express the action of the polarizers at the detection stations and that $|\phi_{\mathbf{n}}\rangle$ and $|\chi_{\mathbf{n}}\rangle$ are simple products and now represent the wavefunction of the separate particles. The original singlet is now split between two different simple product bra-kets. And we see in the second step of the A and B functions that we have quaternions defined from the first step. Even though the measurement functions look deterministic, they are not since QM cannot predict the individual event by event outcomes. A plot of those outcomes after proper analysis does not produce the negative cosine curve. The probability remains at fifty percent for either $+1$ or -1 results (up or down). The cross products in the functions do not contribute to that probability but they do contribute to the $-\mathbf{a}\cdot\mathbf{b}$ probability via the product of the functions. That is easy to see in the simulation provided in the supplemental materials. Since the

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limit replacement functions emulate the action of the detection polarizers, over many events, the average of the action should be correct at about fifty percent.

Note that the measurement functions represent simultaneous detection processes occurring at two possibly spacelike separated observation stations of Alice and Bob. Although occurring simultaneously, $A(\mathbf{a},\, \mathbf{s_1})$ and $B(\mathbf{b},\, \mathbf{s_2})$ are independent physical processes that are *not* subject to the conservation of the initial zero spin angular momentum. Before proceeding with the product calculation, the k indices will be suppressed after the first step. Upon using the "product of limits equal to limits of product" rule, leads to the expectation value calculated as follows [7]:

$$
\begin{aligned}
\mathcal{E}(\mathbf{a},\, \mathbf{b}) \;&=\; \lim_{n>>1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} A(\mathbf{a}^k,\, \mathbf{s}_1^k)\; B(\mathbf{b}^k,\, \mathbf{s}_2^k)\right] && (13)\\
&= \lim_{n\,\gg\,1}\left\{\frac{1}{n}\sum_{k\,=\,1}^{n}\left[\lim_{\mathbf{s}_1\,\to\,\mu_a}\{\,\mathbf{q}(\eta_{\mathbf{as}_1},\, \mathbf{r}_a)\}\right]\left[\lim_{\mathbf{s}_2\,\to\,\mu_b}\{\,\mathbf{q}(\eta_{\mathbf{s}_2\mathbf{b}},\, \mathbf{r}_b)\}\right]\right\} && (14)\\
&= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\lim_{\substack{\mathbf{s}_1\,\to\,\mu_a\\ \mathbf{s}_2\,\to\,\mu_b}}\Big\{\mathbf{q}(\eta_{\mathbf{as}_1},\, \mathbf{r}_a)\;\mathbf{q}(\eta_{\mathbf{s}_2\mathbf{b}},\, \mathbf{r}_b)\Big\}\right] && (15)\\
&= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\lim_{\substack{\mathbf{s}_1\,\to\,\mu_a\\ \mathbf{s}_2\,\to\,\mu_b}}\Big\{[\cos(\eta_{\mathbf{as}_1}) + (I_3\mathbf{r}_a)\,\sin(\eta_{\mathbf{as}_1})]\,[\cos(\eta_{\mathbf{s}_2\mathbf{b}}) + (I_3\mathbf{r}_b)\,\sin(\eta_{\mathbf{s}_2\mathbf{b}})]\Big\}\right] && (16)\\
&= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\lim_{\substack{\mathbf{s}_1\,\to\,\mu_a\\ \mathbf{s}_2\,\to\,\mu_b}}\Big\{-\,\mathbf{q}(\eta_{\mathbf{ab}},\, \mathbf{r}_0)\Big\}\right] && (17)\\
&= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\lim_{\substack{\mathbf{s}_1\,\to\,\mu_a\\ \mathbf{s}_2\,\to\,\mu_b}}\Big\{-\cos(\,\eta_{\mathbf{ab}}) - (I_3\mathbf{r}_0)\sin(\,\eta_{\mathbf{ab}})\Big\}\right] && (18)\\
&= \;-\cos(\,\eta_{\mathbf{ab}}) - \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n}\Big(I_3\,\vec{\mathbf{0}}\Big)\;\sin(\,\eta_{\mathbf{ab}})\right] && (19)\\
&= -\cos(\,\eta_{\mathbf{ab}}) + 0 && (20)\\
&= -\mathbf{a}\cdot\mathbf{b}, && (21)\\
\text{with}\quad \mathbf{r}_0(\mathbf{s}_1,\, \mathbf{s}_2) &= \frac{(\mathbf{a}\cdot\mathbf{s}_1)(\mathbf{s}_2\times\mathbf{b}) + (\mathbf{s}_2\cdot\mathbf{b})(\mathbf{a}\times\mathbf{s}_1) - (\mathbf{a}\times\mathbf{s}_1)\times(\mathbf{s}_2\times\mathbf{b})}{\sin(\eta_{\mathbf{ab}})}. && (22)
\end{aligned}
$$

Here in step (16) we have used the cosine and sine representation of the quaternions along with the geometric algebra pseudo-scalar, $I_3 = e_1e_2e_3$. When that is expanded and calculated out using geometric algebra we get step (17), see [6, 7]. In other words, the product of two quaternions is another quaternion and $\mathbf{r}_0$ happens to have all cross products in the numerator that are zero when the limits are taken. So, in step (19), the limit replacement functions act only on the cross products of $\mathbf{r}_0$ and produce a null vector. Thus we obtain the correct result via a completely local process. So, we can see here that QM is completed by 3-sphere topology discussed in [6, 7] via quaternions. And that the singlet has a parallelized 3-sphere topology that is passed on to the particle pair [5–7]. This is very easy to demonstrate via the Pauli algebra.

We have verified the above analytical calculation via a computer simulation using the programming language of Mathematica, which is presented in the supplemental materials as a PDF file [8]. The Mathematica notebook file is also available at [9]; see also [10].

Acknowledgments

The author gives many thanks to Joy Christian and Jay Yablon who contributed much to this paper via many discussions.

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[8] See Supplemental Material at [URL will be inserted by publisher] for the Mathematica programming code of the simulation as a PDF file and the Mathematica notebook file.

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