Simulation Based on Joy Christian's original 3-Sphere Model.
Parts of Quaternion Code by John Reed.
Modified by Fred Diether Dec. 2021. With 3D Vectors!

Load Quaternion Package, Set Run Time Parameters, Initialize Arrays and Tables

```
<< Quaternions`
β0 = Quaternion[1, 0, 0, 0];
β1 = Quaternion[0, 1, 0, 0];
β2 = Quaternion[0, 0, 1, 0];
β3 = Quaternion[0, 0, 0, 1];
Qcoordinates = {β1, β2, β3};
Qcoordinates2 = {β0, β1, β2, β3};
m = 30 000;
Ls1 = ConstantArray[0, m];
Ls2 = ConstantArray[0, m];
Da1 = ConstantArray[0, m];
Db1 = ConstantArray[0, m];
qa1 = ConstantArray[0, m];
qb1 = ConstantArray[0, m];
outA = Table[{0, 0}, m];
outB = Table[{0, 0}, m];
a1 = ConstantArray[0, m];
b1 = ConstantArray[0, m];
```

Generating Particle Data with Three Independent Do-Loops

In[312]:=
```
Do[s = RandomPoint[Sphere[]];  (*Singlet 3D vector*)  (*Hidden Variable*)
 Ls1[[i]] = s.Qcoordinates;  (*Convert to quaternion coordinates*)
 Ls2[[i]] = -s.Qcoordinates, {i, m}]
```

In[313]:=
```
Do[a = RandomPoint[Sphere[]];  (*Detector 3D vector angle*)
 a1[[i]] = a;
 Da = a.Qcoordinates;  (*Convert to quaternion coordinates*)
 Da1[[i]] = Da;
 qa = Da ** Ls1[[i]];
 qa1[[i]] = qa;
 qA = Re[Da ** Limit[Ls1[[i]], Ls1[[i]] → Sign[Re[Da ** Ls1[[i]]]] Da]];
 outA[[i]] = {a, qA}, {i, m}]
```

In[314]:=
```
Do[b = RandomPoint[Sphere[]];  (*Detector 3D vector angle*)
 b1[[i]] = b;
 Db = b.Qcoordinates;  (*Convert to quaternion coordinates*)
 Db1[[i]] = Db;
 qb = Ls2[[i]] ** Db;
 qb1[[i]] = qb;
 qB = Re[Db ** Limit[Ls2[[i]], Ls2[[i]] → Sign[Re[Db ** Ls2[[i]]]] Db]];
 outB[[i]] = {b, qB}, {i, m}]
```

VERIFICATION OF THE ANALYTICAL 3-SPHERE MODEL BASED ON GEOMETRIC ALGEBRA USING QUATERNIONS

In[315]:=
```
q = 0; t = 0; u = 0;
r0 = ConstantArray[0, m];
QAB = ConstantArray[0, m];
plotq = Table[{0, 0}, m];
A = outA[[All, 2]];
B = outB[[All, 2]];
```

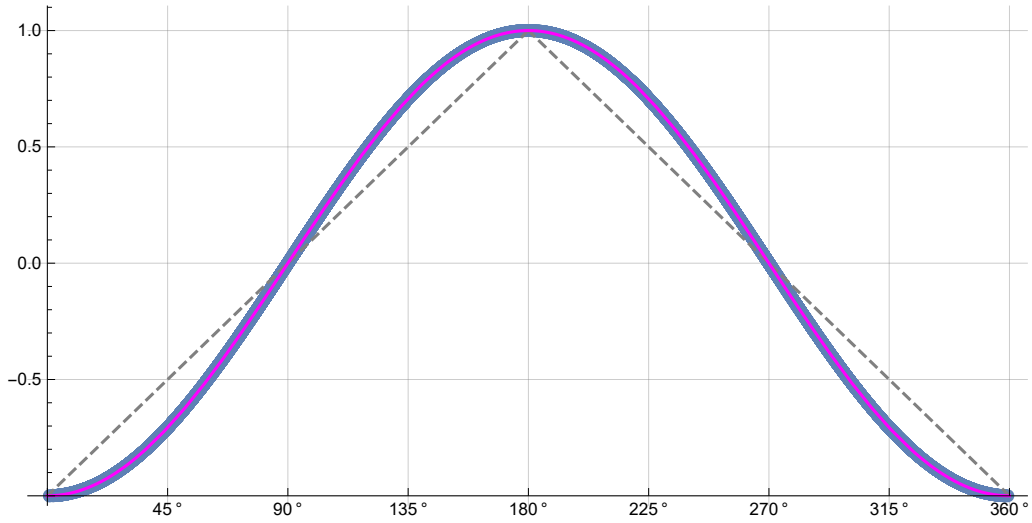
In[321]:=
```
Do[QAB[[i]] = Re[Da1[[i]] ** Ls1[[i]] ** Ls2[[i]] ** Db1[[i]]];
r0[[i]] = (Re[qa1[[i]]] Limit[Cross[s2, b1[[i]]], s2 → Sign[Re[qb1[[i]]]] b1[[i]]] +
     Re[qb1[[i]]] Limit[Cross[a1[[i]], s1], s1 → Sign[Re[qa1[[i]]]] a1[[i]]] -
     Limit[Cross[a1[[i]], s1], s1 → Sign[Re[qa1[[i]]]] a1[[i]]] *
      Limit[Cross[s2, b1[[i]]], s2 → Sign[Re[qb1[[i]]]] b1[[i]]]) /
   (Sin[ArcCos[a1[[i]].b1[[i]]]]);
 q = {Re[QAB[[i]]], r0[[i]][[1]], r0[[i]][[2]], r0[[i]][[3]]}.Qcoordinates2;
 t = t + A[[i]];
 u = u + B[[i]];
 AveA = t / m;
 AveB = u / m;
 ϕA = ArcTan[a1[[i]][[1]], a1[[i]][[2]]] / 50;
 ϕB = ArcTan[b1[[i]][[2]], b1[[i]][[1]]] / 50;
 If[ϕA * ϕB > 0, θ = ArcCos[a1[[i]].b1[[i]]] * 180 / π,
  θ = (2 π - ArcCos[a1[[i]].b1[[i]]]) * 180 / π];
 plotq[[i]] = {θ, Re[q]}, {i, m}]
Print["<A> = ", AveA]
Print["<B> = ", AveB]
meanq = Mean[plotq[[All, 2]]];
Print["Imaginary part vanishes. meanq = ", meanq]
sim = ListPlot[plotq, PlotMarkers → {Automatic, Small},
   AspectRatio → 8 / 16, Ticks → {{{0, 0 °}, {45, 45 °}, {90, 90 °}, {135, 135 °},
       {180, 180 °}, {225, 225 °}, {270, 270 °}, {315, 315 °}, {360, 360 °}}, Automatic},
   GridLines → Automatic, AxesOrigin → {0, -1.0}];
p1 = Plot[-1 + 2 x Degree / π, {x, 0, 180}, PlotStyle → {Gray, Dashed}];
p2 = Plot[3 - 2 x Degree / π, {x, 180, 360}, PlotStyle → {Gray, Dashed}];
negcos = Plot[-Cos[x Degree], {x, 0, 360}, PlotStyle → {Magenta}];
Show[sim, p1, p2, negcos]
```

<A> = −0.005

<B> = −0.0048

Imaginary part vanishes. meanq = −0.000280495

Out[330]=



Blue is the data, magenta is the negative cosine curve for an exact match.